MPM Simulation of Fracture Including Both Crack Tip Fracture Mechanics and Fiber Bridging Traction Laws - Application to Fracture of Composites and Medium Density Fiber Board (MDF)

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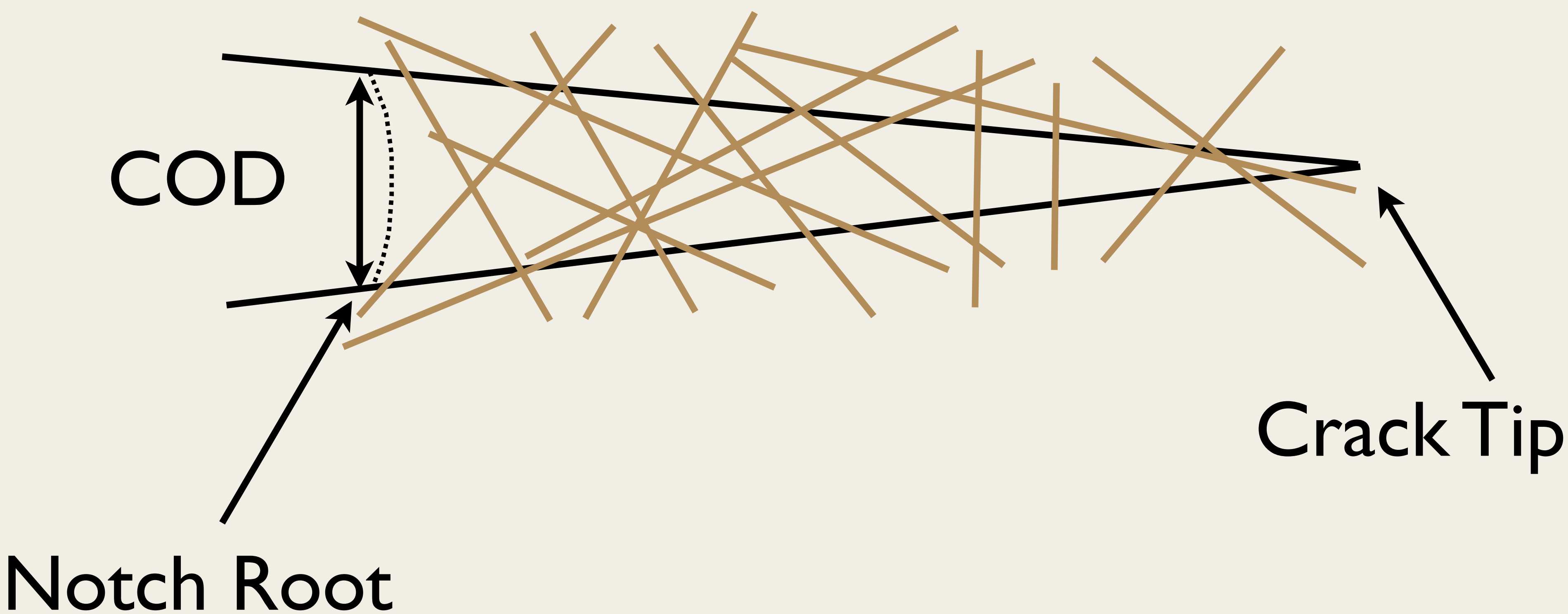
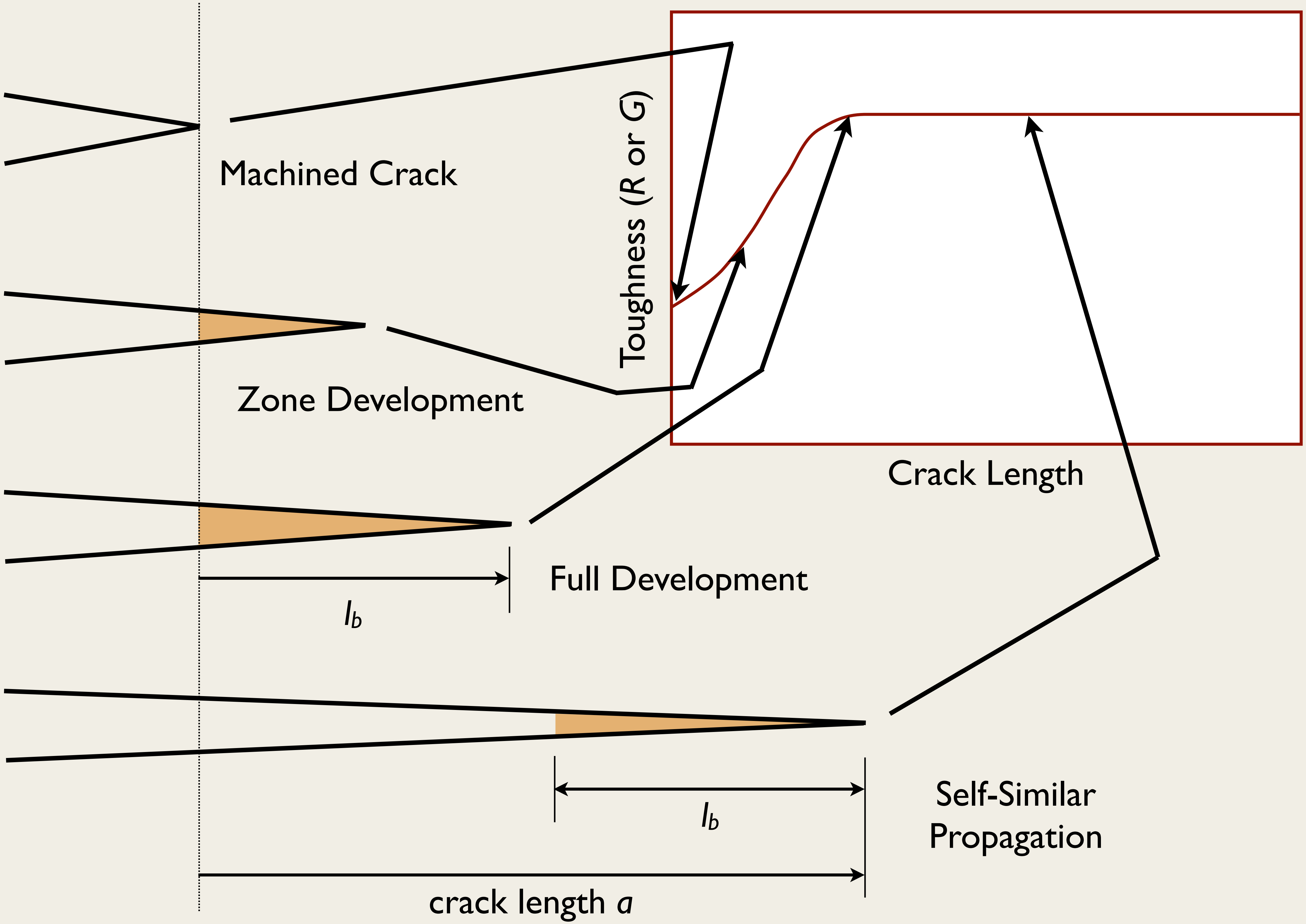
Fracture Process Zone

Fibrous Composites
Wood
Concrete

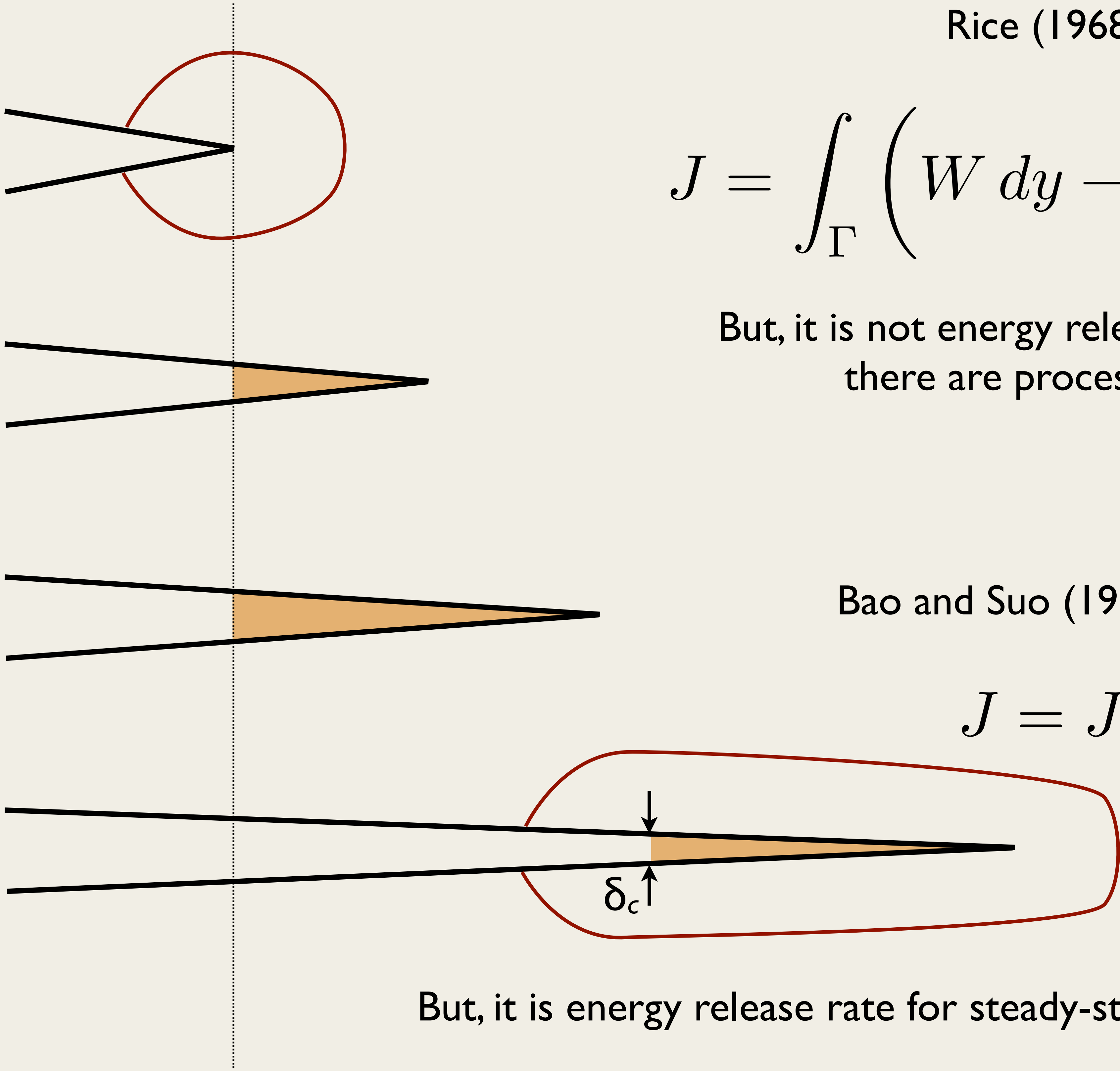


Note: there are two "crack tips"

Fracture Mechanics R Curve



Fracture Mechanics *J* Integral



Rice (1968)

$$J = \int_{\Gamma} \left(W\, dy - \vec{T} \cdot \frac{d\vec{u}}{dx} dx \right)$$

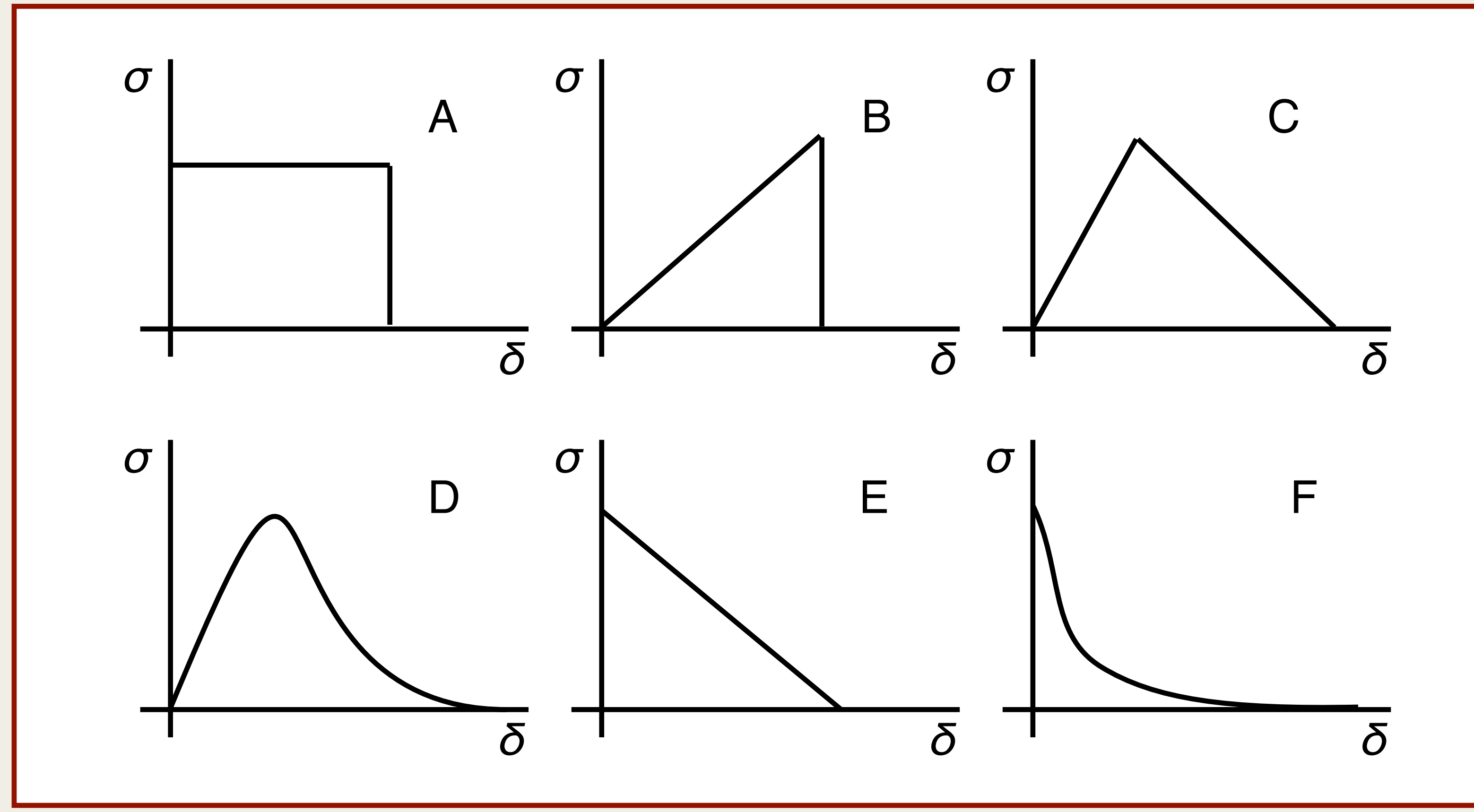
But, it is not energy release rate when there are process zones

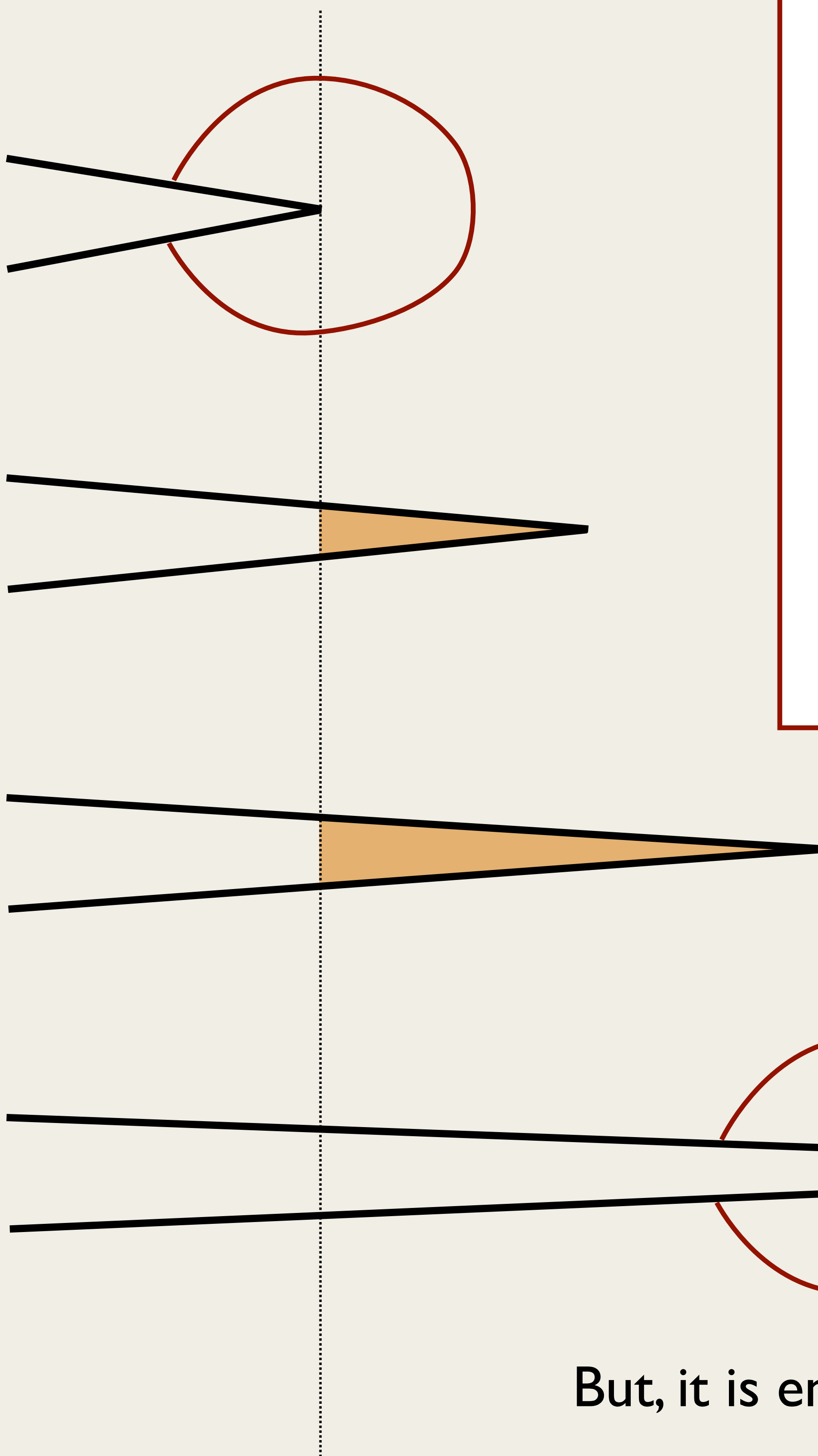
Bao and Suo (1992)

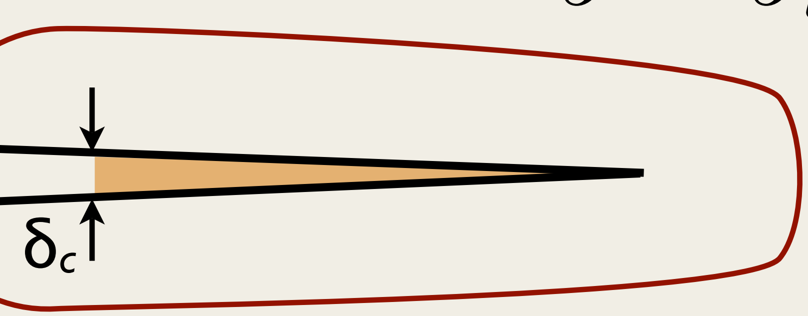
$$J = J_{tip} + \int_{0}^{\delta_c} \sigma(\delta)\, d\delta$$

But, it is energy release rate for steady-state propagation only

Fracture Mechanics *J* Integral







Bao and Suo (1992)

$$J = J_{tip} + \int_0^{\delta_c} \sigma(\delta)\, d\delta$$

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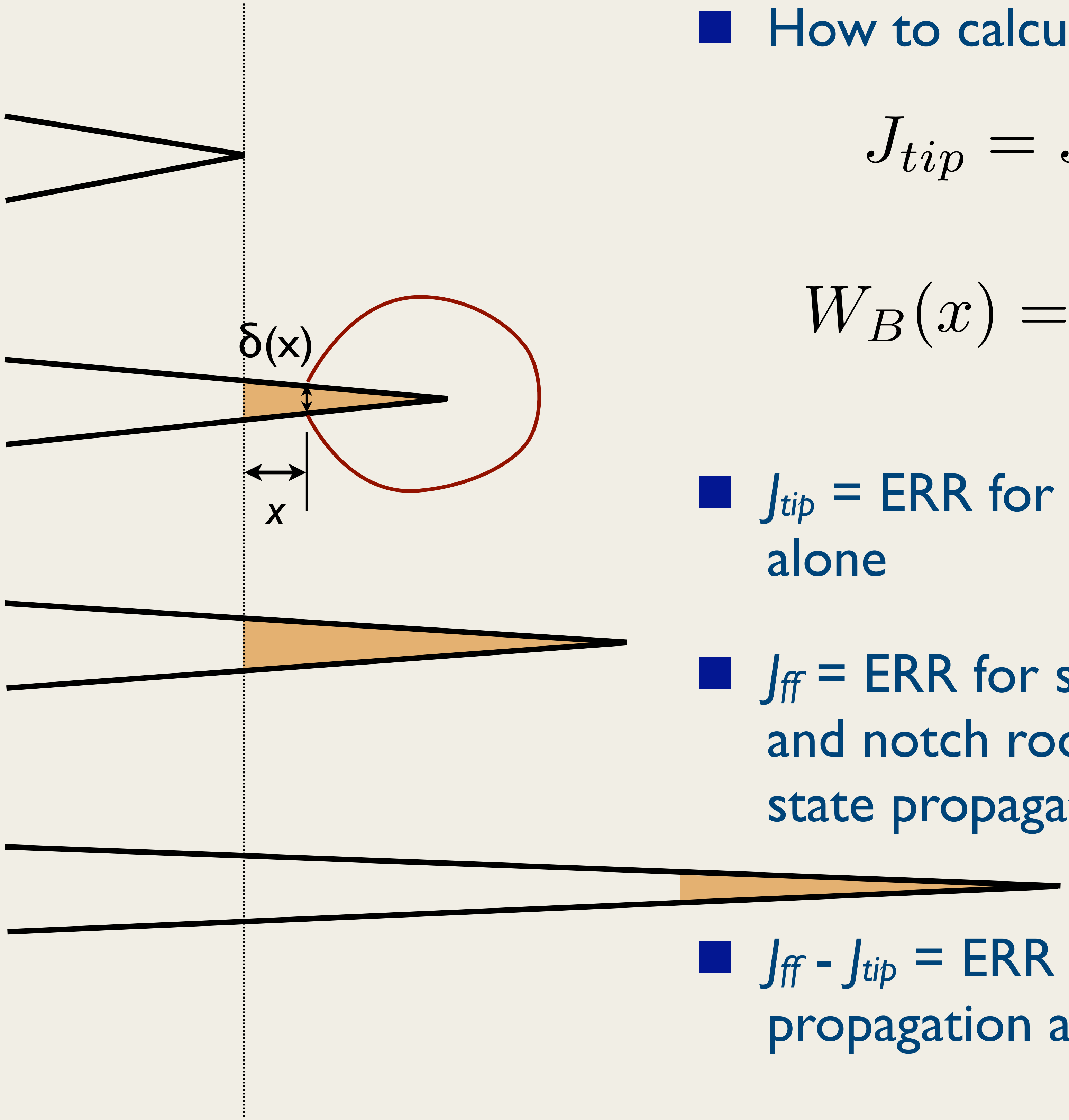
Tools for Analysis of Rising R Curve



- How to calculate J_{tip}?

$$J_{tip} = J(x) - W_B(x)$$

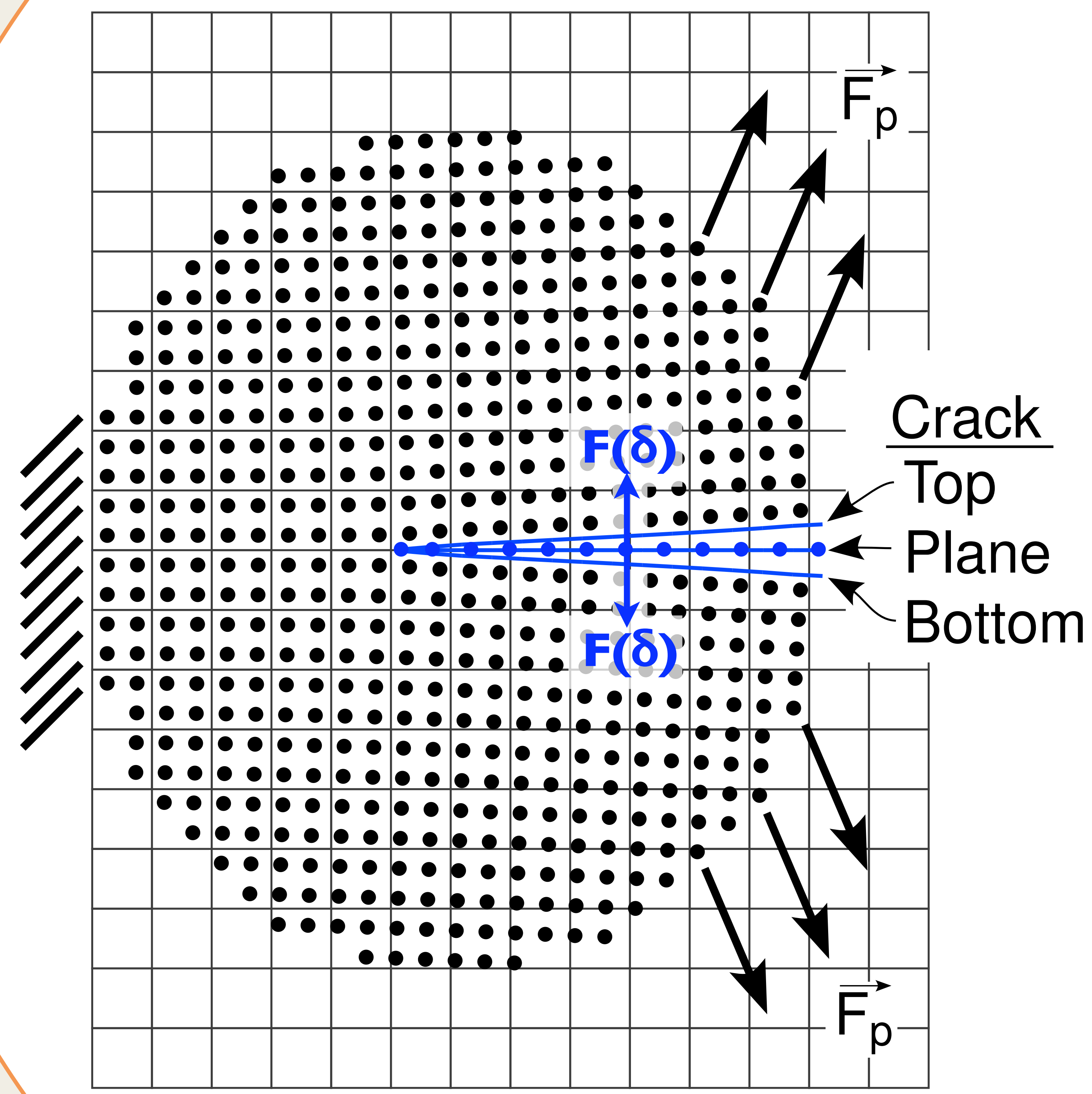
$$W_B(x) = \int_0^{\delta(x)} \sigma(\delta)\, d\delta$$

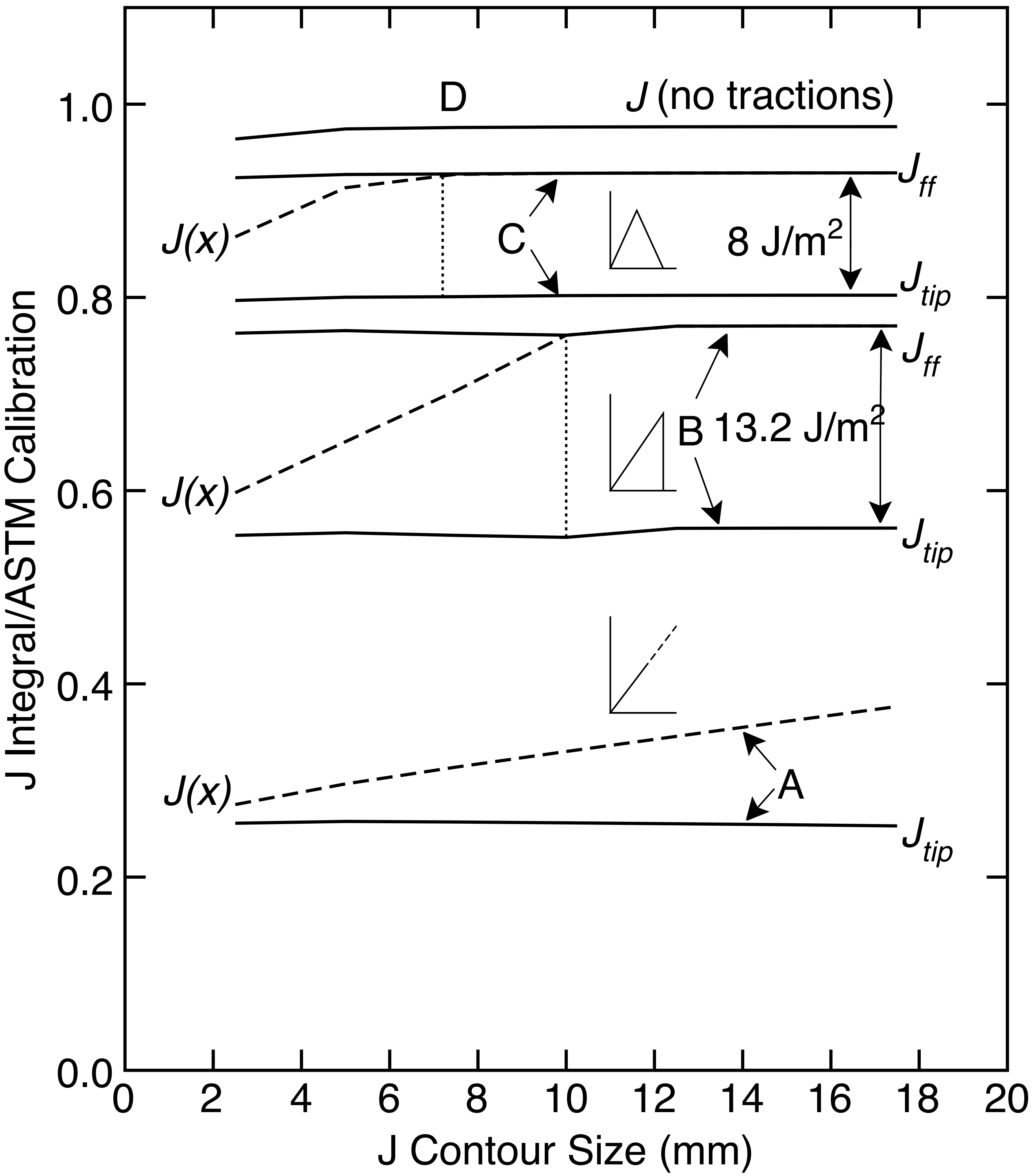
- J_{tip} = ERR for crack tip propagation alone
- J_{ff} = ERR for simultaneous crack tip and notch root propagation (steady-state propagation)
- J_{ff} - J_{tip} = ERR for notch root propagation alone.

J_{tip} and J_{ff} in MPM

- Explicit Crack in MPM - CRAMP
  - John A. Nairn, "Material Point Method Calculations with Explicit Cracks," *Computer Modeling in Eng. & Sci.*, **4**, 649-664 (2003).
- J Integral Calculation in MPM
  - Yajun Guo and John A. Nairn, "Calculation of J-Integral and Stress Intensity Factors using the Material Point Method," *Computer Modeling in Eng. & Sci.*, **6**, 295-308 (2004).
- Imperfect Interfaces in MPM
  - John A. Nairn, "Numerical Implementation of Imperfect Interfaces," *Computational Materials Science*, **40**, 525-536 (2007).
- Now Combined to Model Fracture Process Zones
  - John A. Nairn, "Analytical and Numerical Modeling of *R* Curves for Cracks with Bridging Zones," *Int. J. Fracture*, in press (2008).

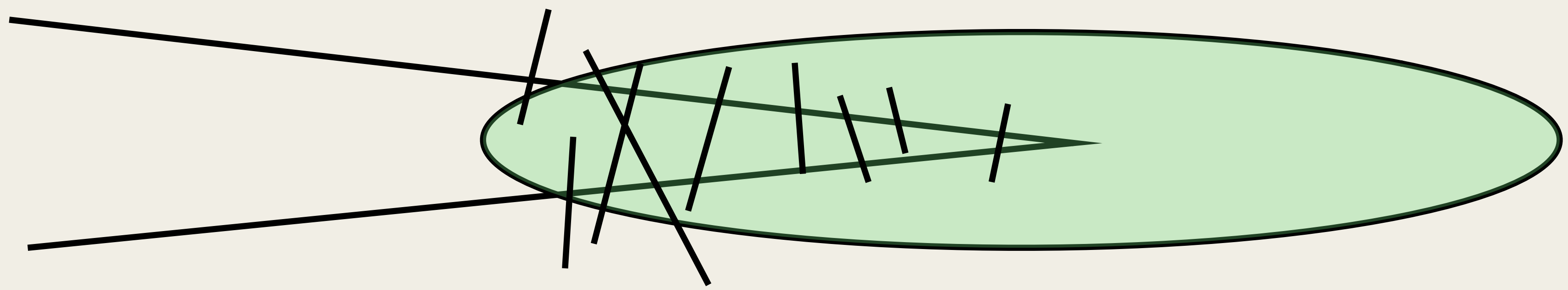
- Crack Particles
- Each Assigned Cohesive "Law"
- Implemented by External Force on the Crack Particle
- History-Dependent Cohesive Law (see later)
- Usual *J* Integral Calculation, but include $W_B(x)$ if needed.



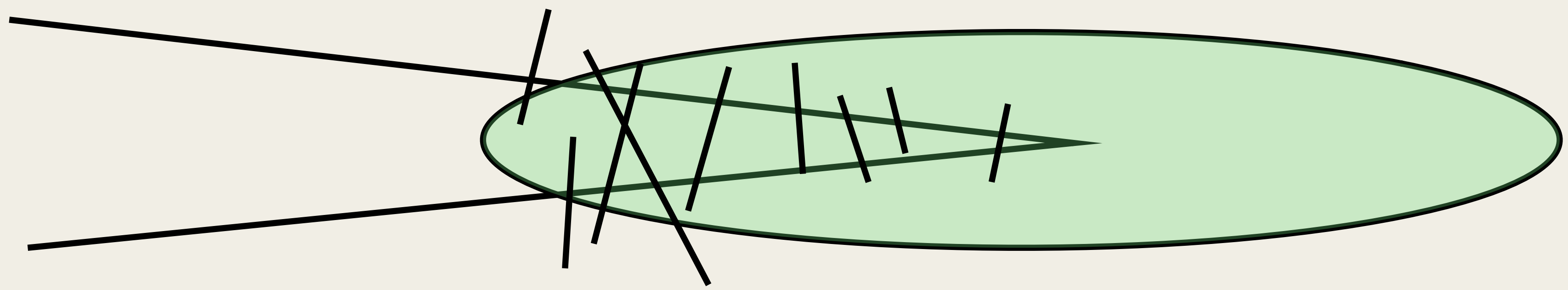


- *J*(*x*) (Rice's integral) is no longer path independent
- J_{tip} and J_{ff} are path independent
- Both can be calculated in MPM with a single contour at any *x*.

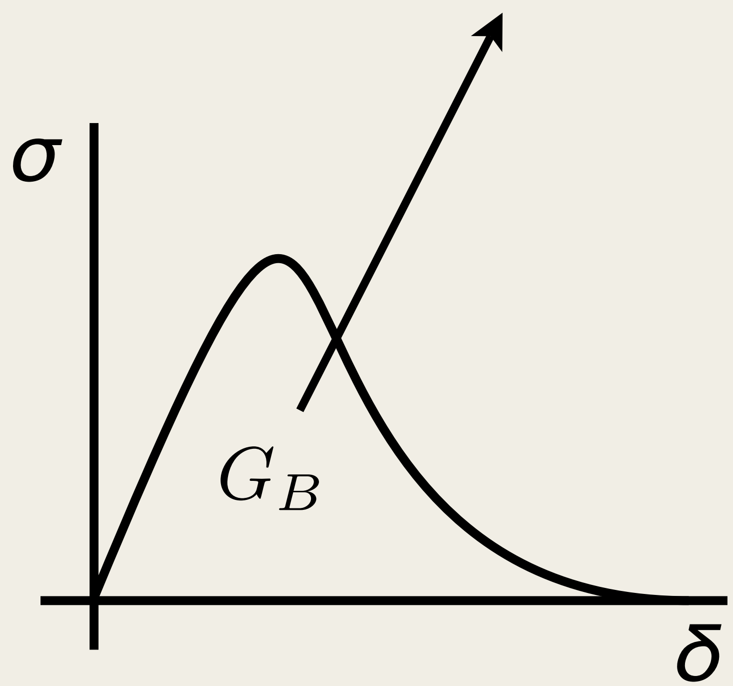
Fiber Bridging Theory

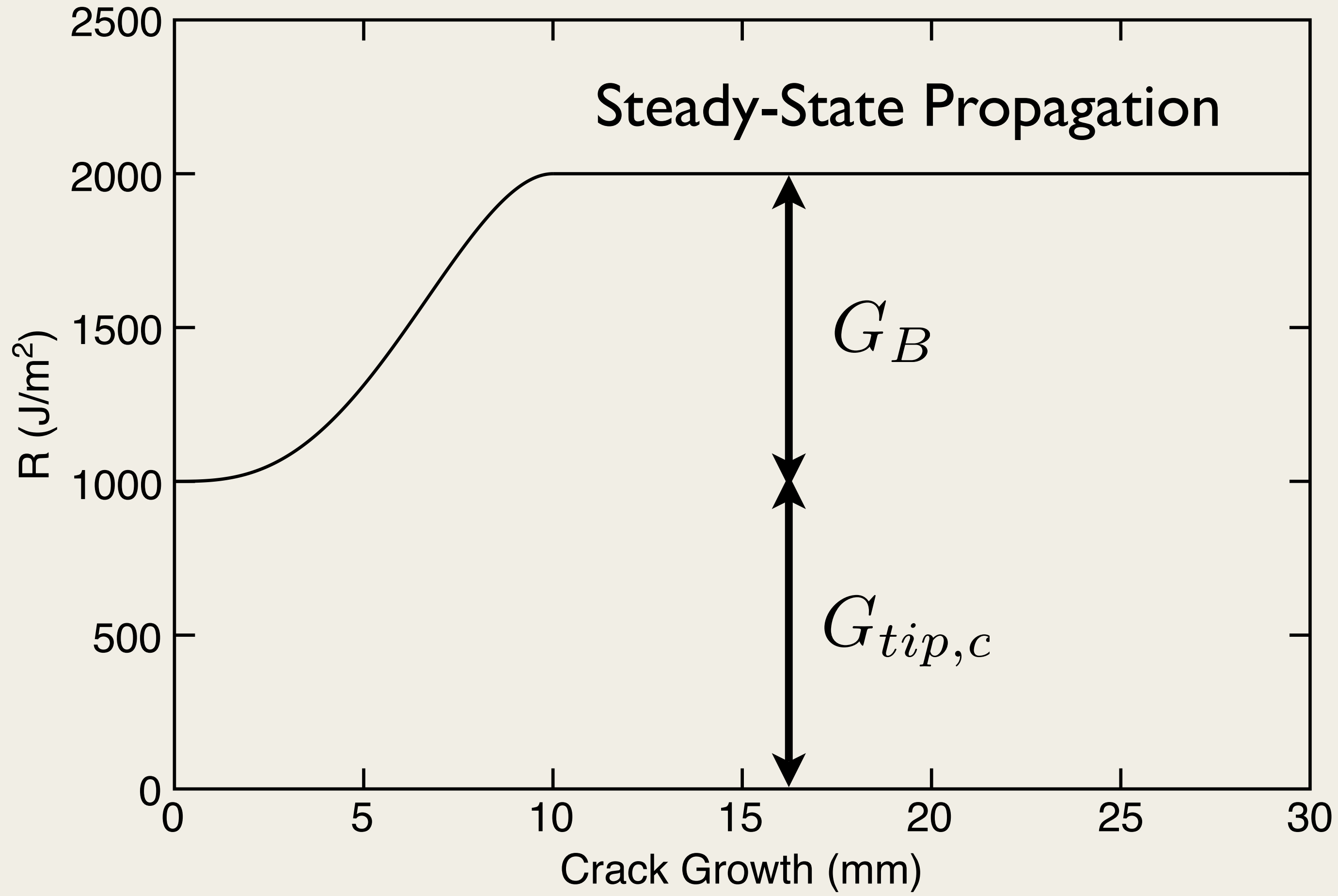


Fiber Bridging Theory



$$R = G_{tip,c} + \int_0^{\delta_c} \sigma(\delta)\, d\delta$$

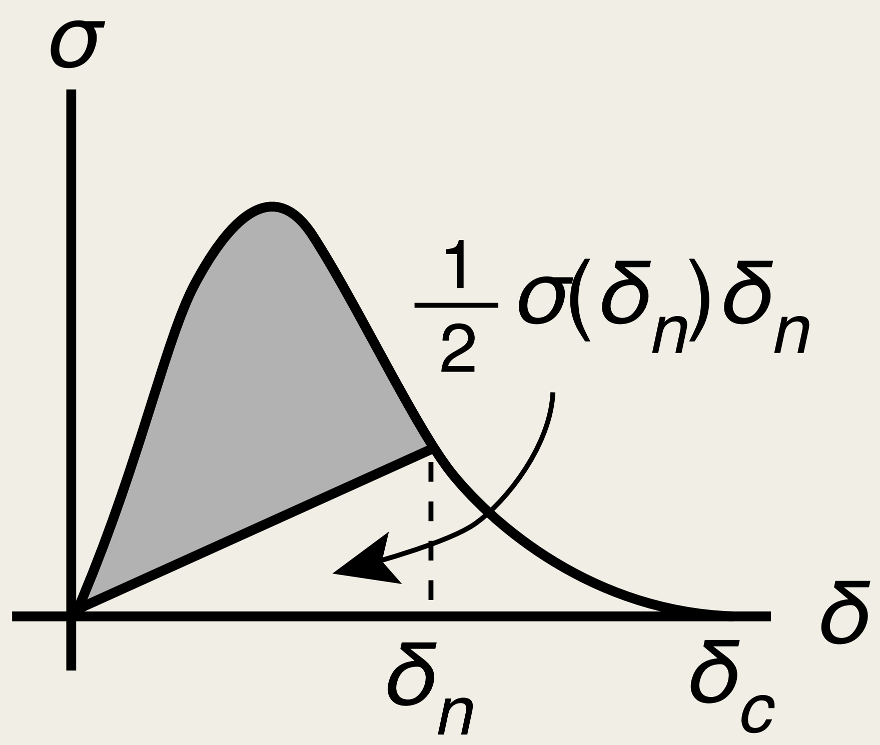


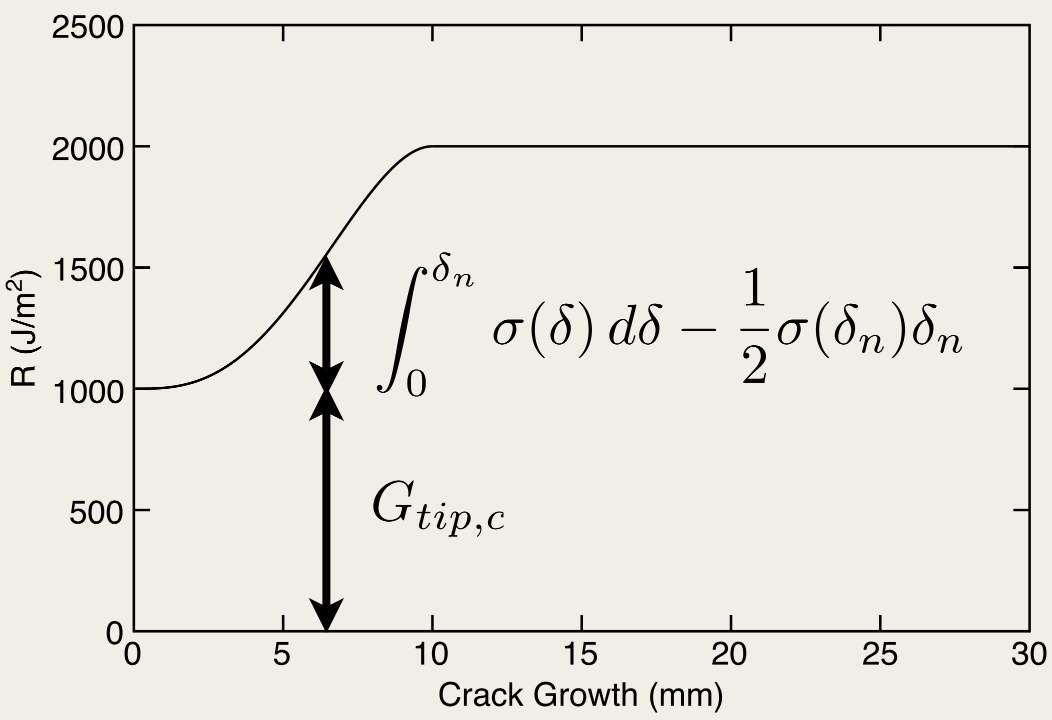


Non Steady State Propagation

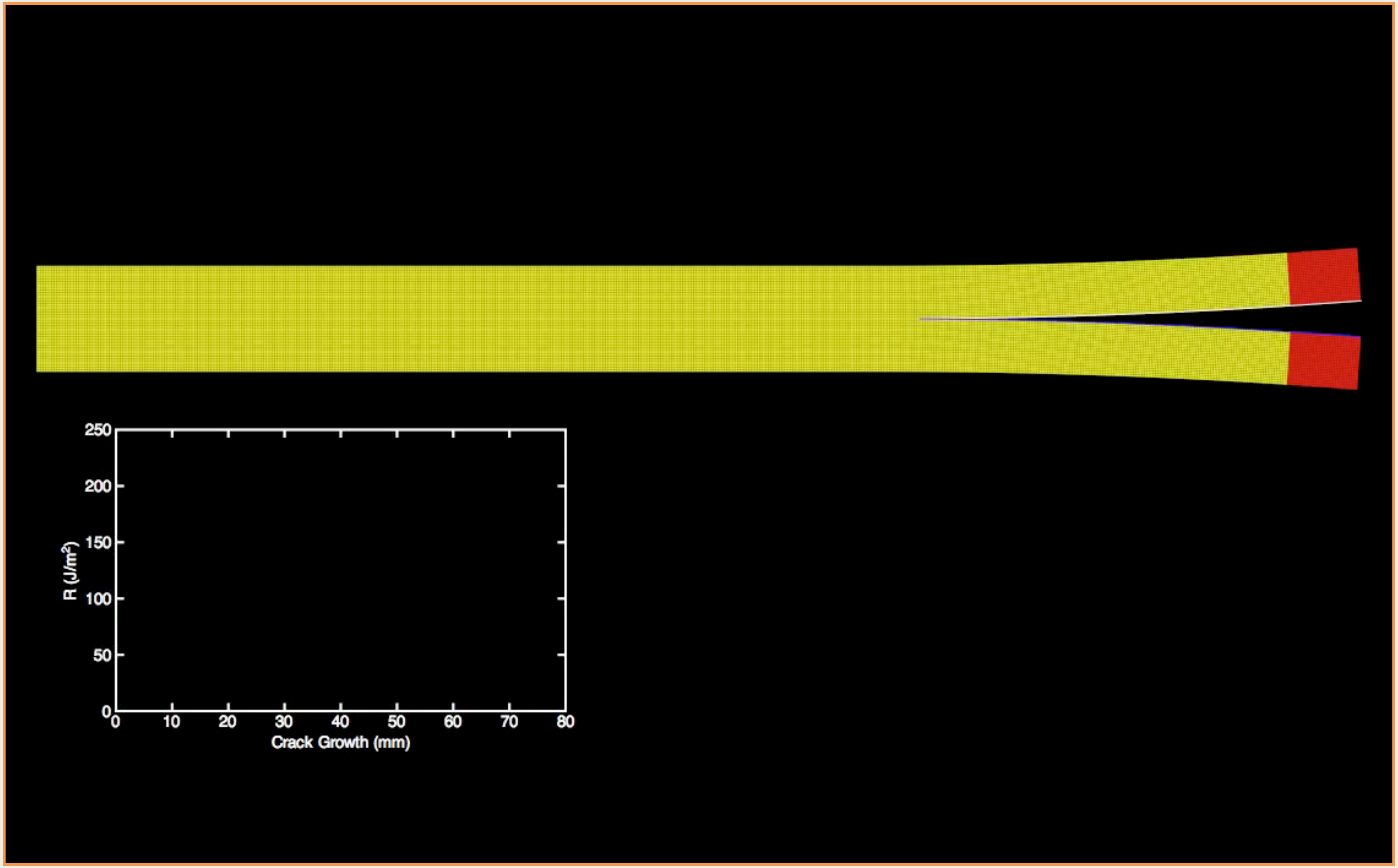
- New Model - Depends on Mechanics of the Process Zone
  - As load increases, calculate J_{tip}, crack propagates when $J_{tip} > J_{tip,c}$
  - Notch root propagates when $\delta_{root} > \delta_c$
  - Before steady state, actual energy release is not given by any J integral. A reasonable model is:

$$R = G_{tip,c} + \int_0^{\delta_n} \sigma(\delta)\, d\delta - \frac{1}{2}\sigma(\delta_n)\delta_n$$



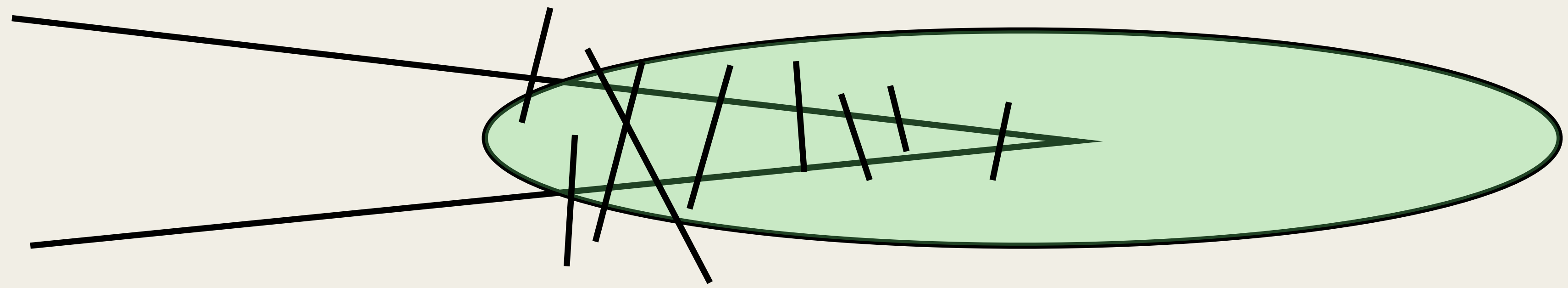


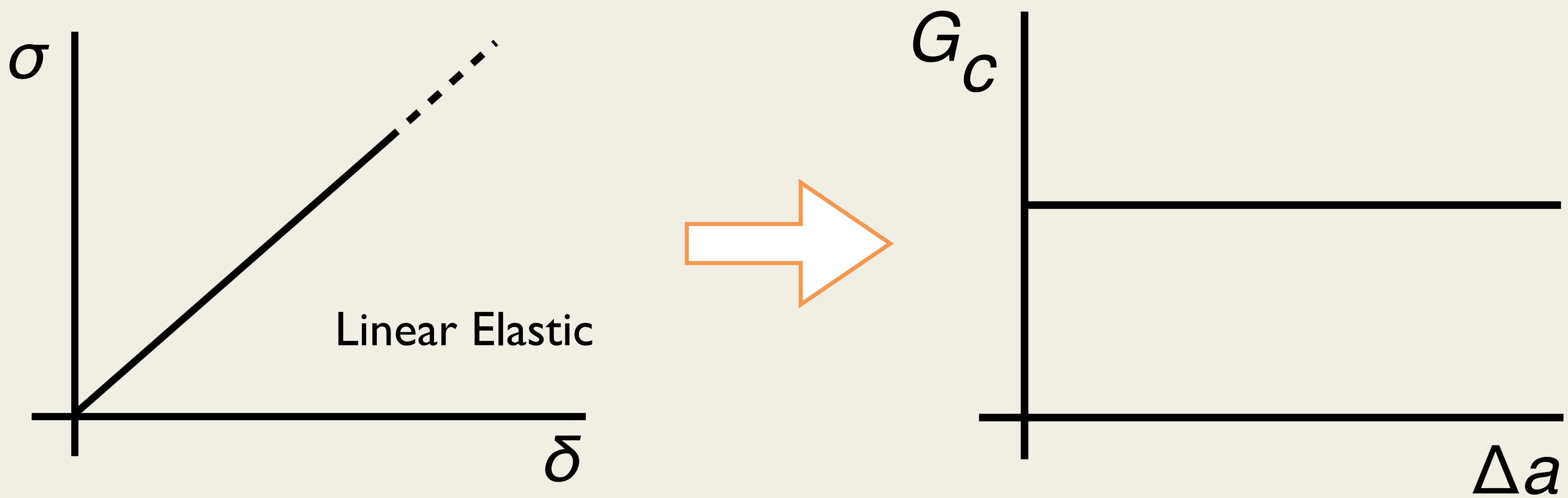
MPM Modeling with Process Zone



Possible Traction "Laws"

- Force-displacement within bridging zone
  - Linear Elastic
  - Elastic/Softening
  - Linear Softening





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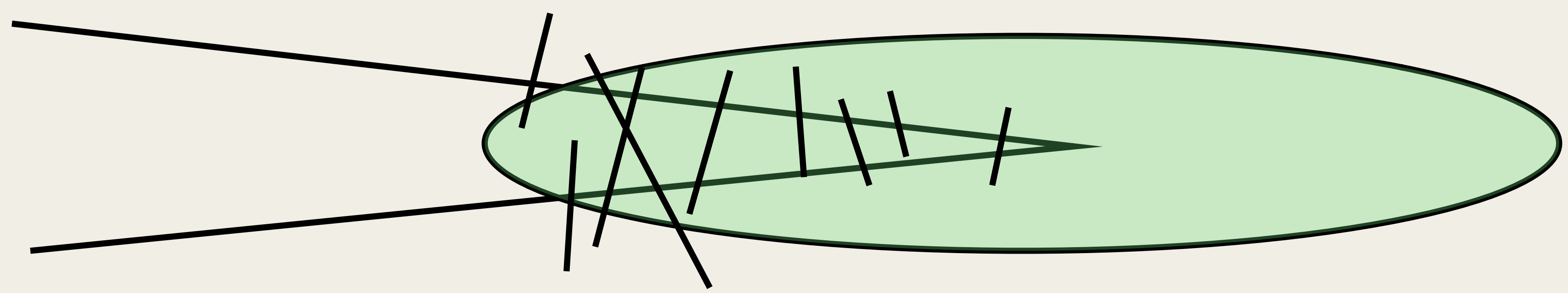

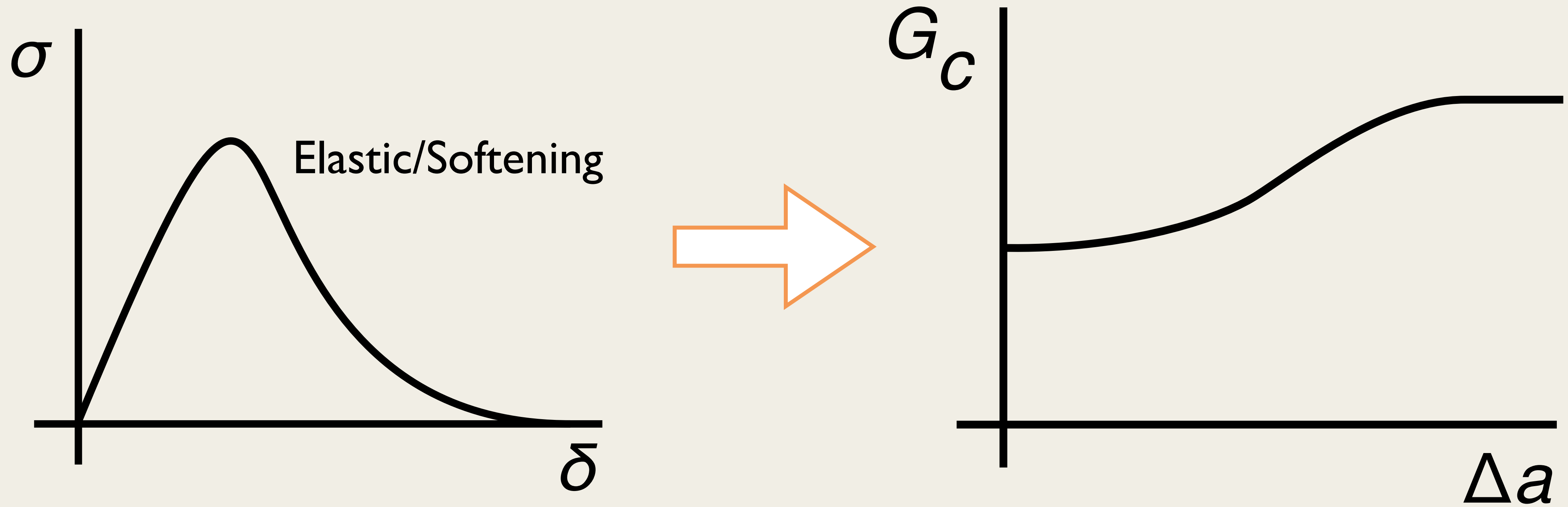

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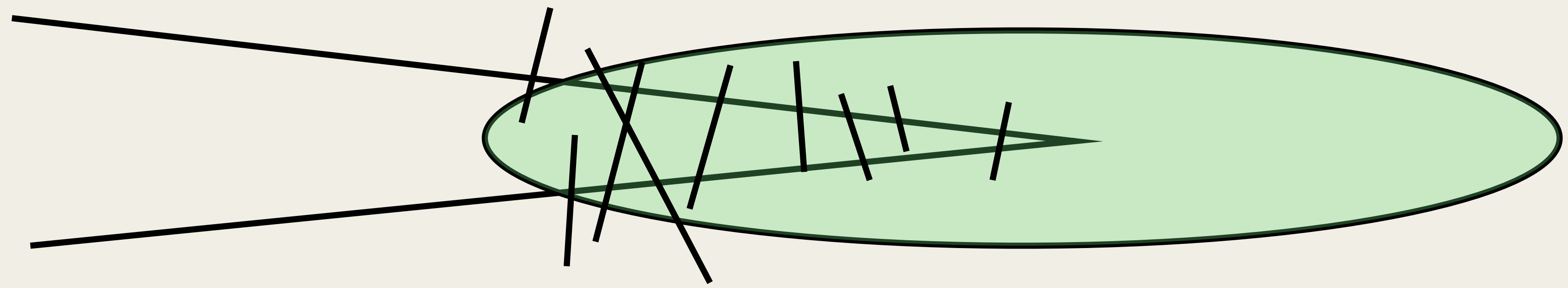
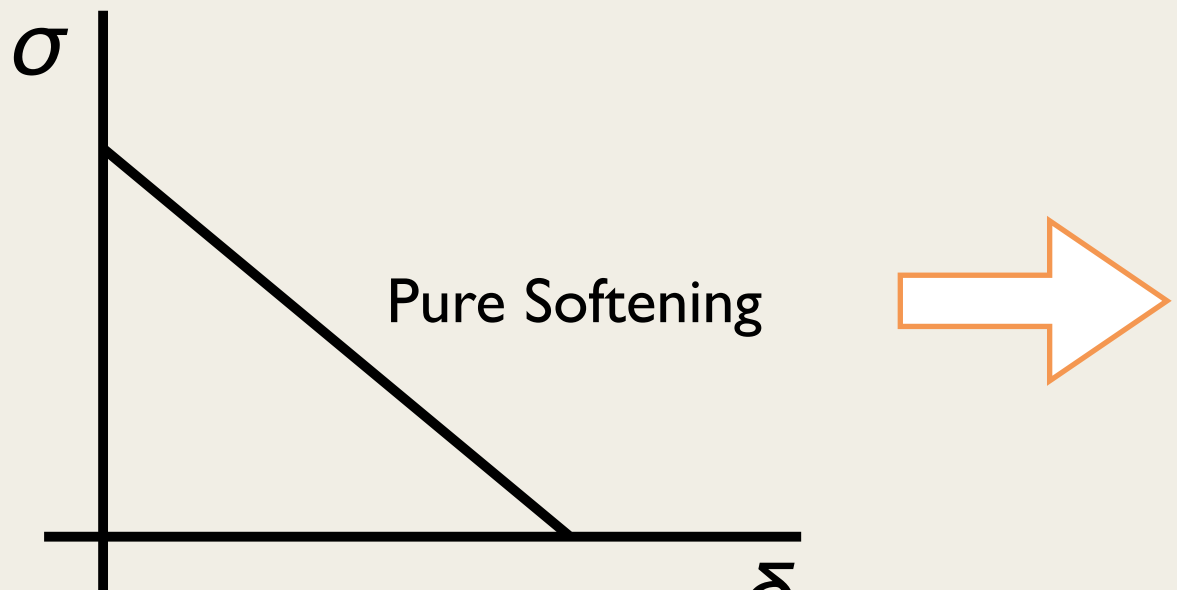
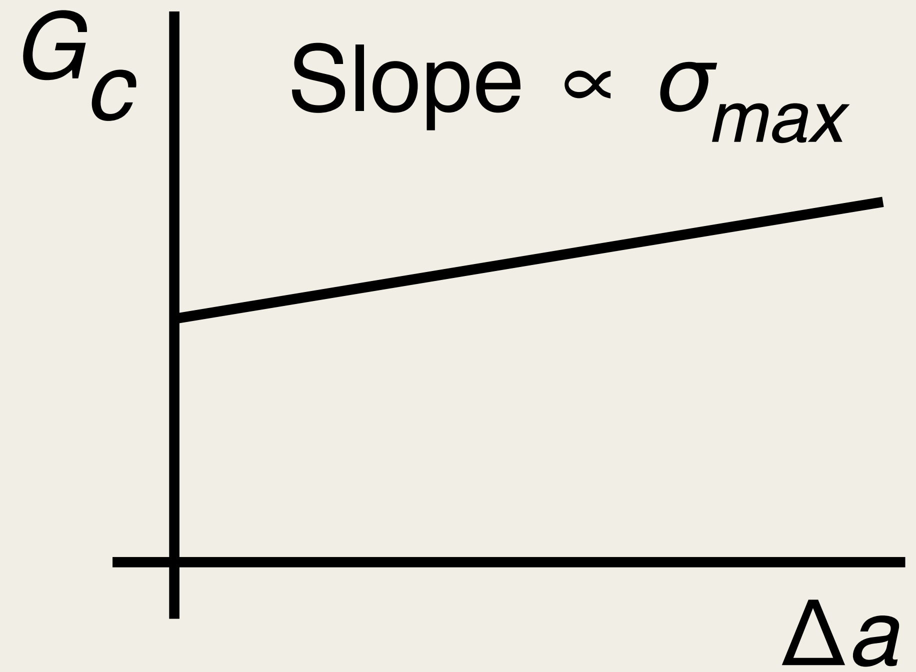
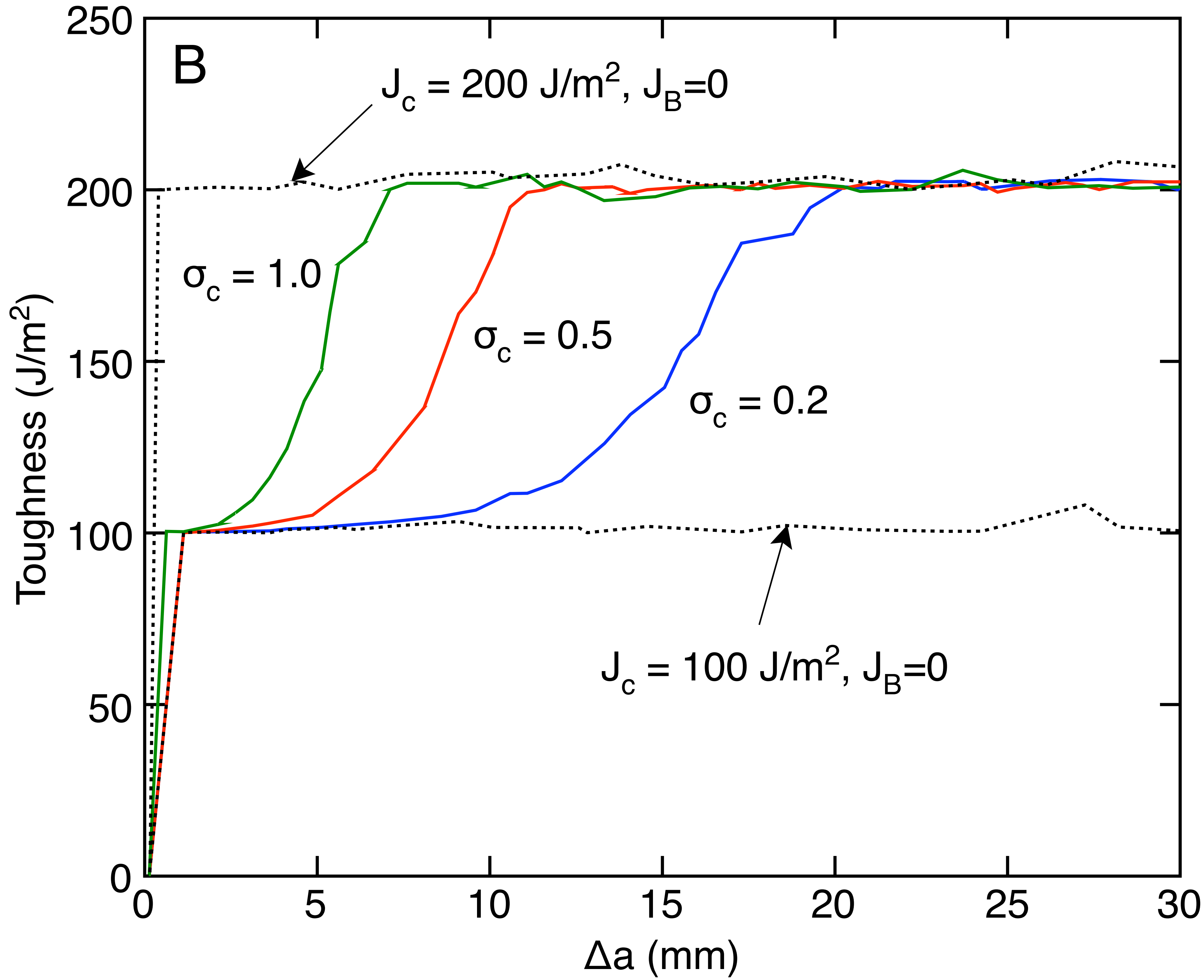




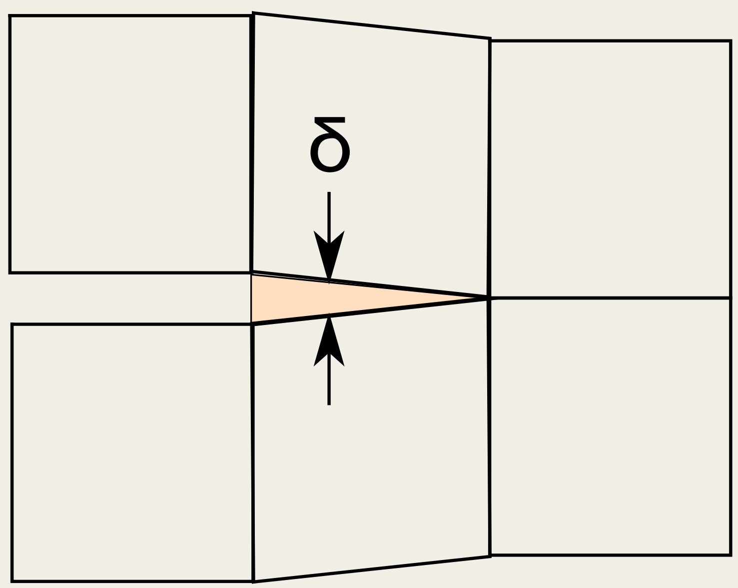


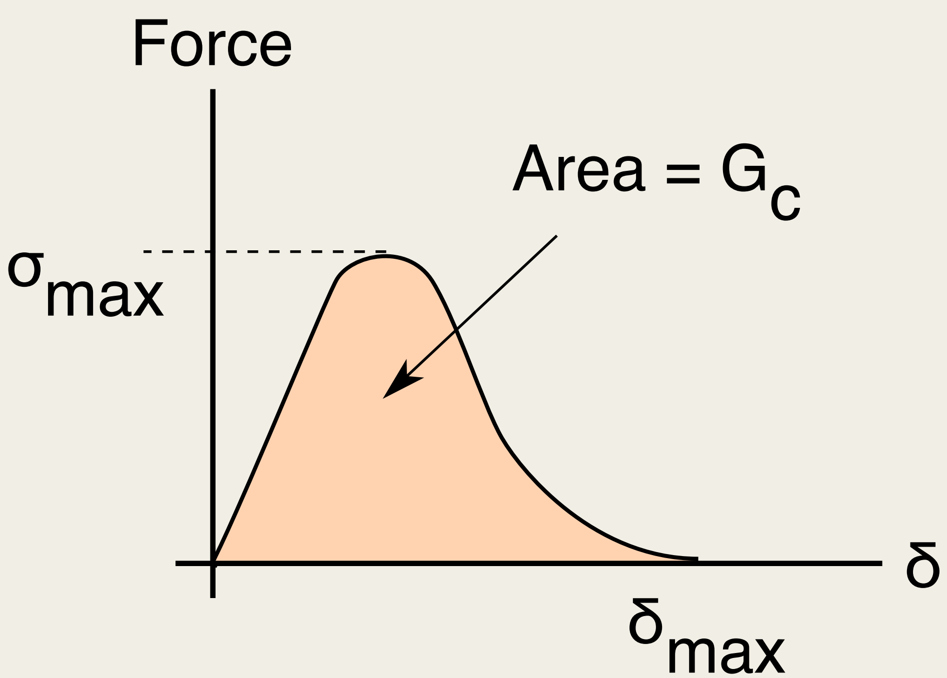
Simulated R Curves *vs.* Fracture Mechanics



Finite Element Cohesive Elements

- Crack Tip Cohesive Element



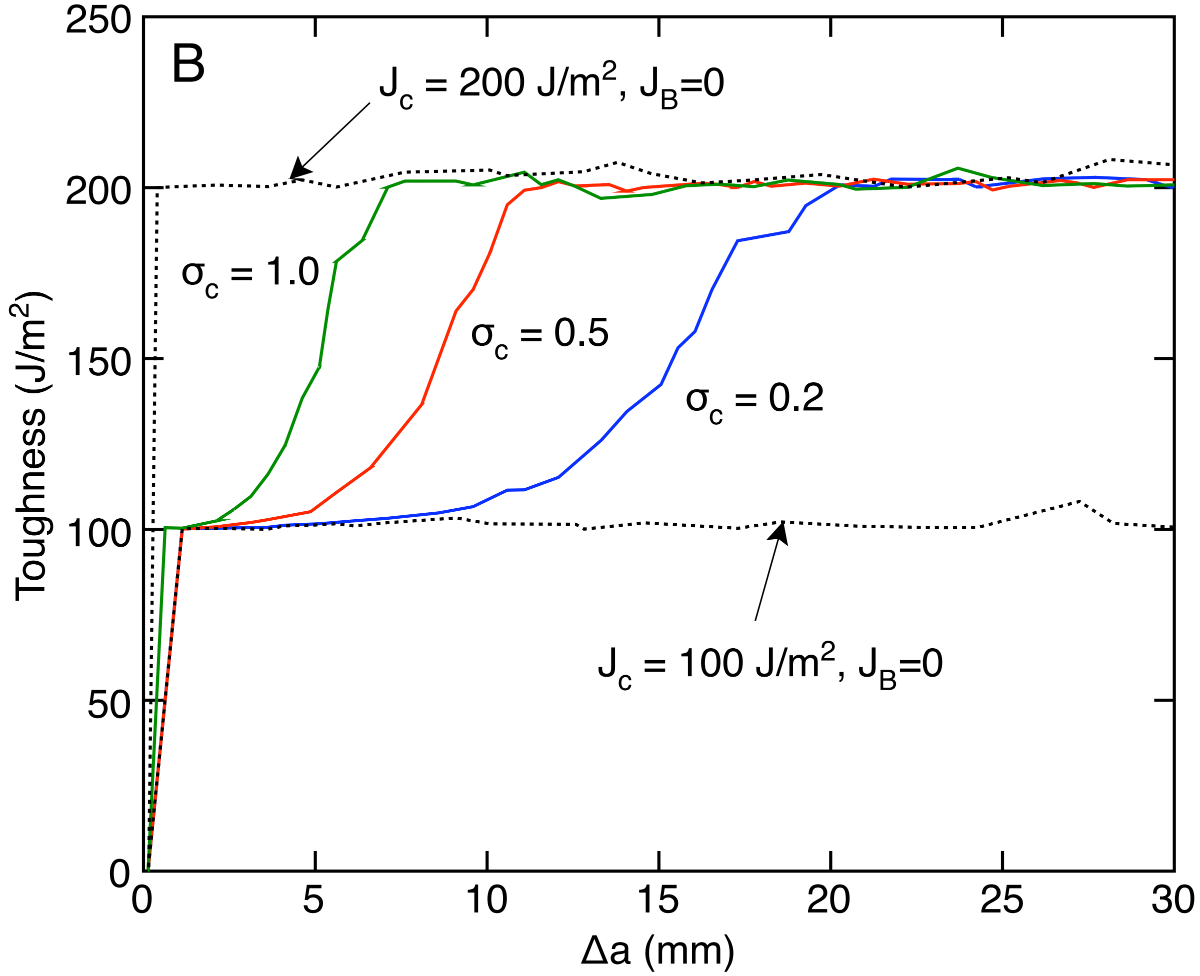


- Originally Called "Fictitious Crack"
- Now exploded into "Cohesive Fracture Modeling"
- Problems?
  - Must know crack path in advance
  - Numerical artifacts caused by pre-inserted cohesive elements
  - Does a "Cohesive Law" even exist? (did researches forget the "fictitious" part?)

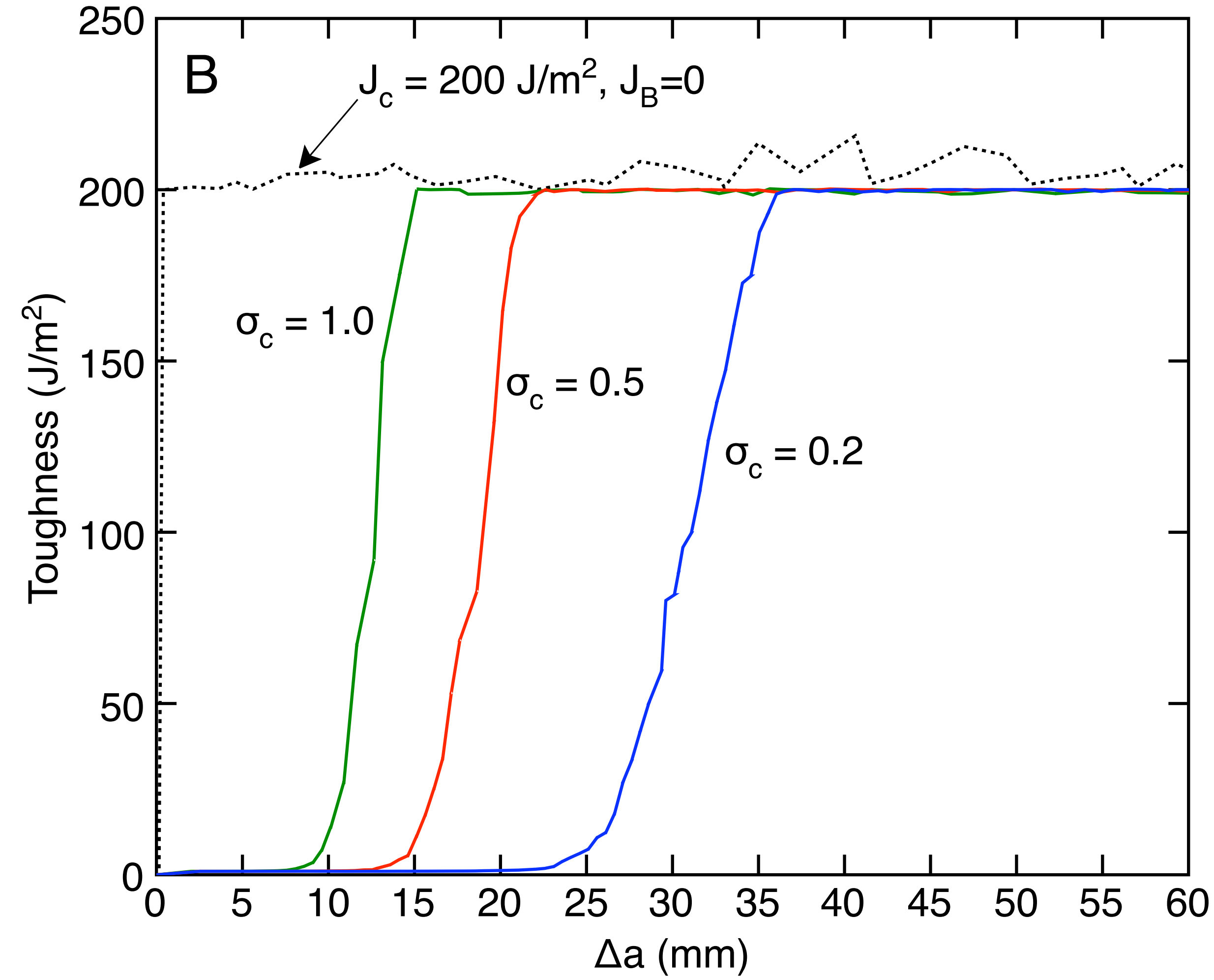
Two Competing Numerical Methods

- Pure Fracture Mechanics
  - No cohesive zone
  - Crack advances by crack-tip energy release rate or stress intensity factor
- Pure Cohesive Zone
  - No singularity, energy release rate is given by the cohesive law
  - Crack advances when "cohesive law" reaches critical COD
- Real Crack Propagation
  - Perhaps reality (as so often happens in science, politics, and life) is between these two extremes and has both crack tip processes and a process zone.
- Implementation in MPM
  - As already described, but now systematically...
  - Vary the crack tip toughness from 100% of steady state toughness (pure fracture mechanics) to 0% (MPM cohesive zone model)

Range of Models

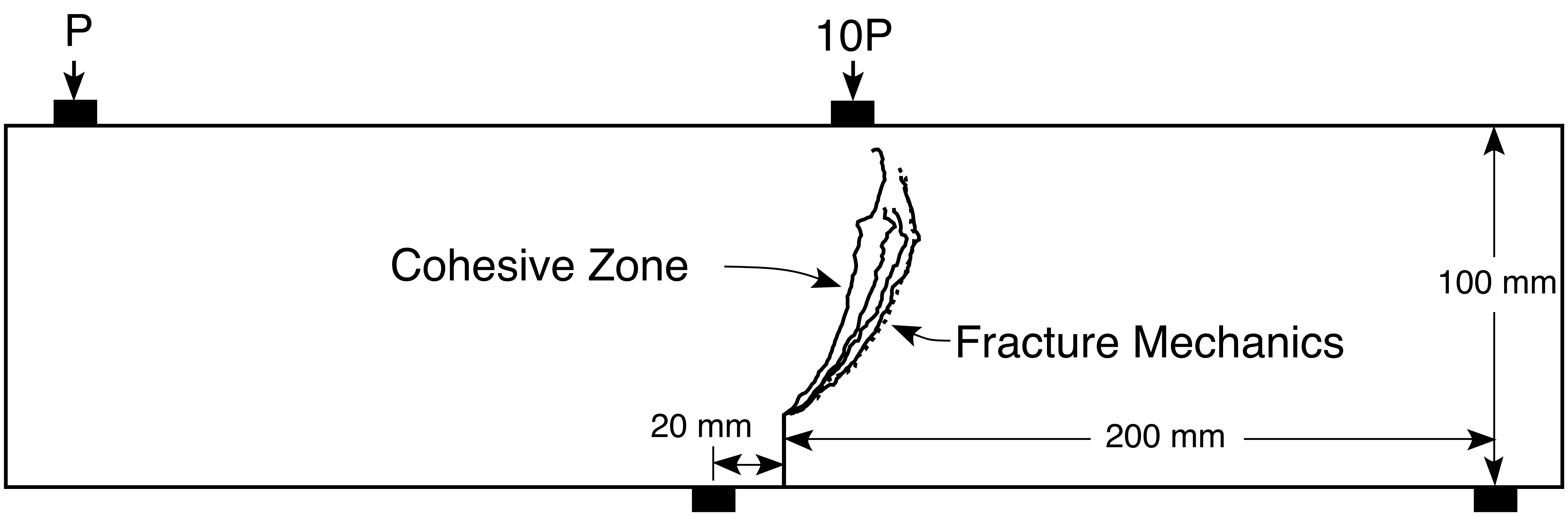


Pure fracture mechanics and
50% fracture mechanics

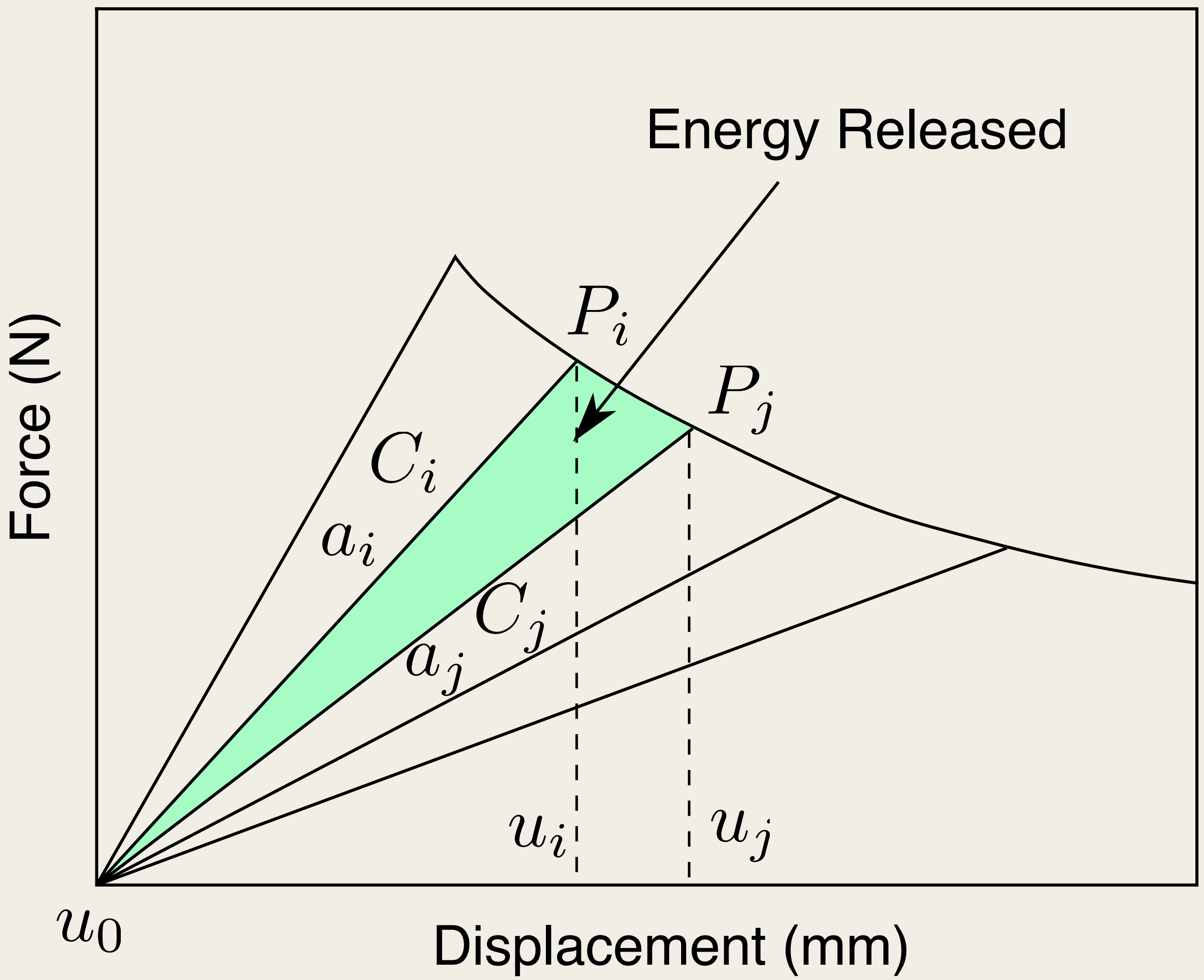


Pure cohesive zone or
0% fracture mechanics

Extension to Unknown Crack Path



Connection to Experimental R Curves



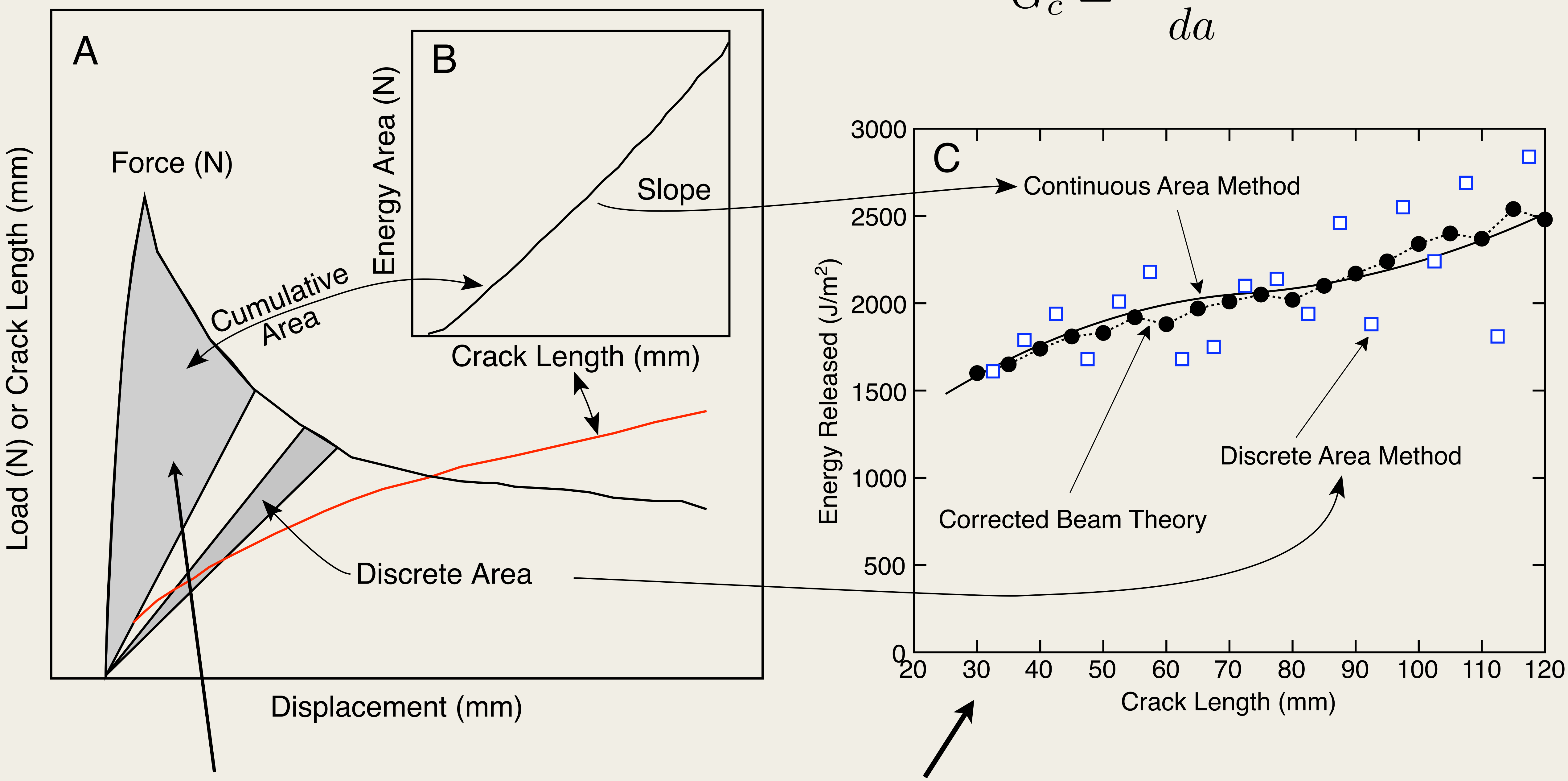
Triangular Area Method

$$G_c = \frac{P_i(u_j - u_0) - P_j(u_i - u_0)}{2B\Delta a_{ij}}$$

- In general there is no way to determine J_{ff} or J_{tip} from experimental data (not always recognized in the literature)
- One can, however, directly measure energy per unit new crack area.
- Experimental Issues
  - Measuring actual crack length
  - Measuring energy incremental area
  - Unloading may change the results (*i.e.*, the process zone is not reversible)

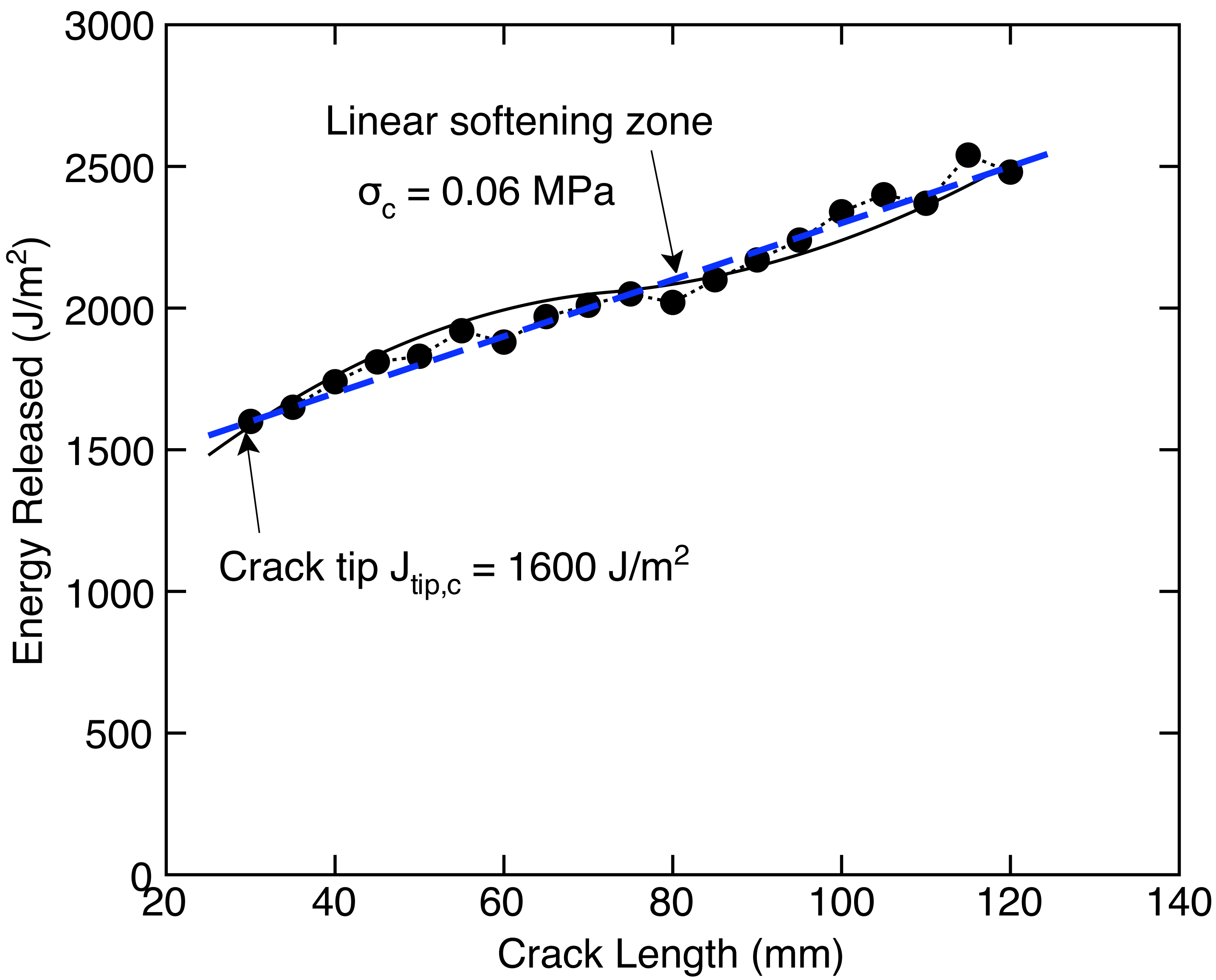
Experimental R Curves Without Unloading

$$G_c = \frac{dU(a)}{da}$$



$$U(d) = \frac{1}{B}\left(\int_0^d F(u)\,du - \frac{1}{2}F(d)d\right)$$

Reanalysis of Hashemi, Kinloch, and Williams (1990)

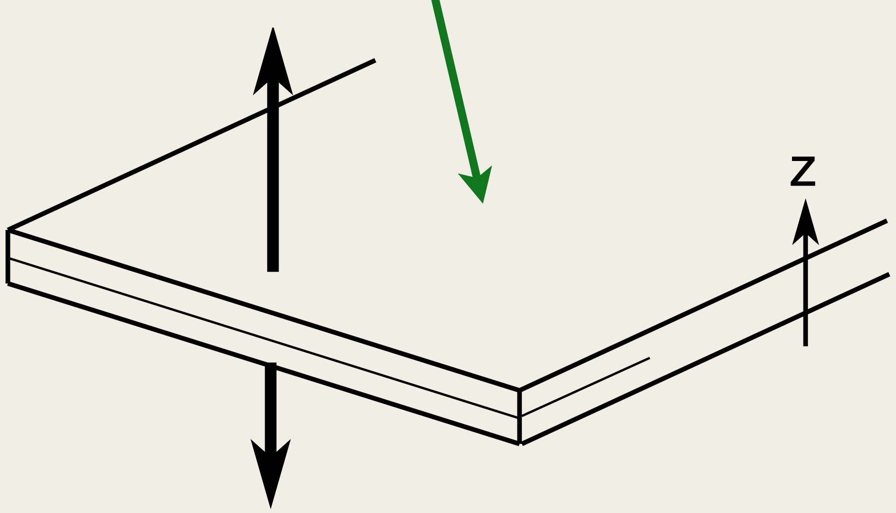


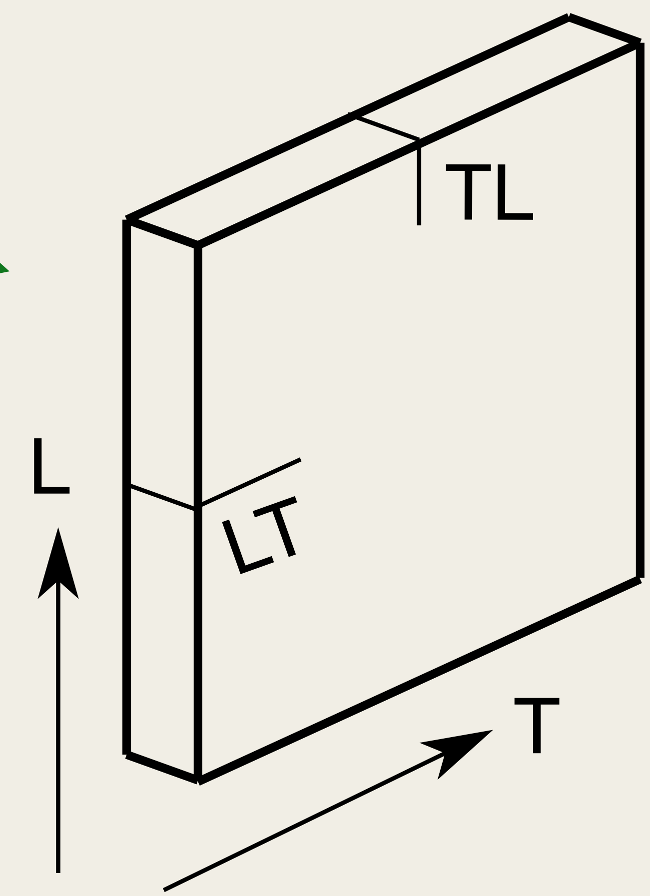
- Hashemi, Kinloch, and Williams
  - Used fracture mechanics with toughness from 1500 to 2500 J/m².
  - Tried cohesive model but needed very high cohesive stress.
  - A physical interpretation of high cohesive stress "is not immediately obvious"
- MPM Model
  - Crack tip toughness of 1600 J/m².
  - Cohesive stress more realistic (0.06 MPa)
  - Entire curve is non-steady state, thus cannot measure bridging toughness

Fracture Toughness of MDF

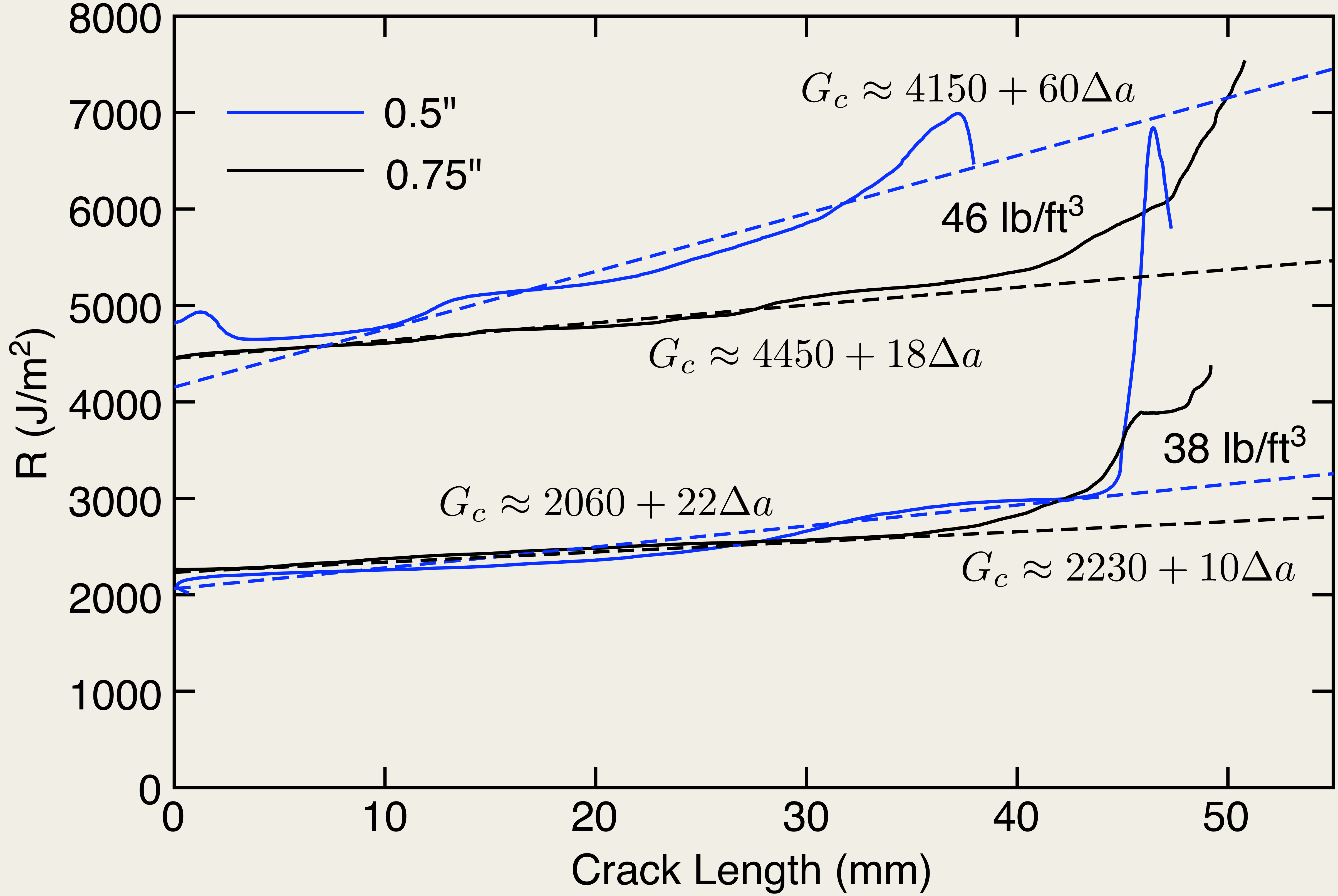
- Medium Density Fiber board (MDF)
  - Wood composite with fine wood fibers bound by resin
  - 3-5 mm softwood fibers or 1-2 mm hardwood fibers

- Measure fracture toughness
  - In-plane cracks
    - Longitudinal (TL)
    - Transverse (LT)
  - Z cracks (ZL or ZT)

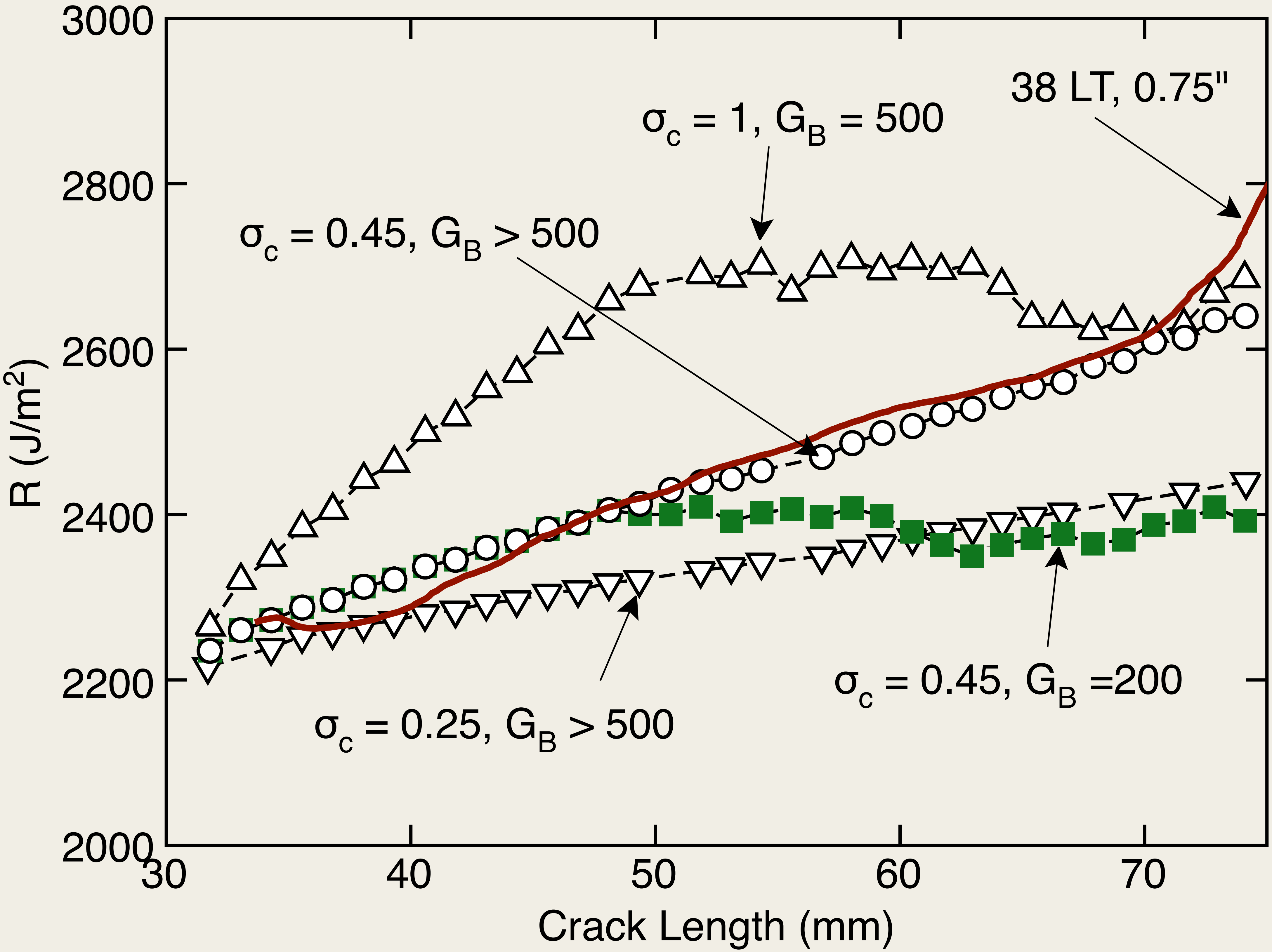




MDF Fracture Resistance Curves



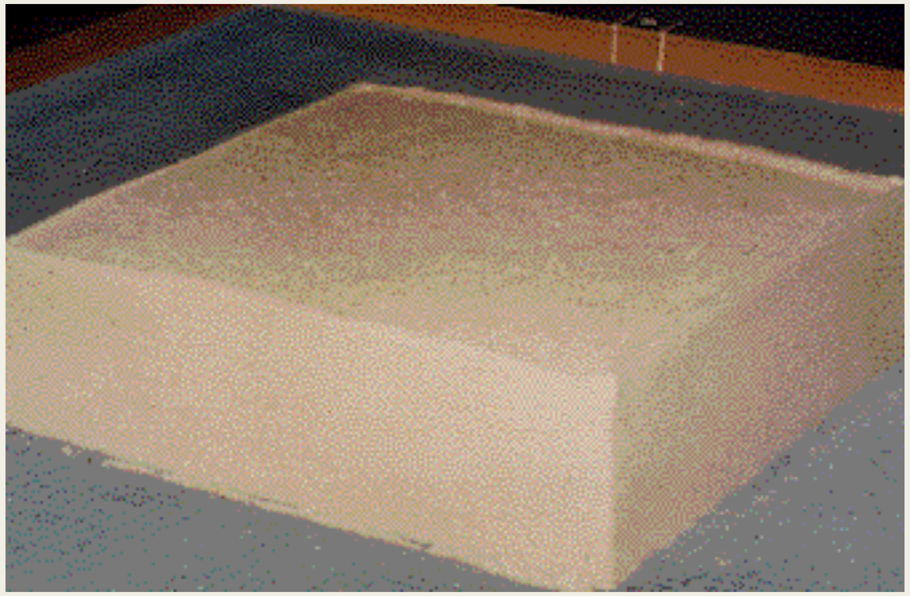
Simulation of MDF Fracture Toughness

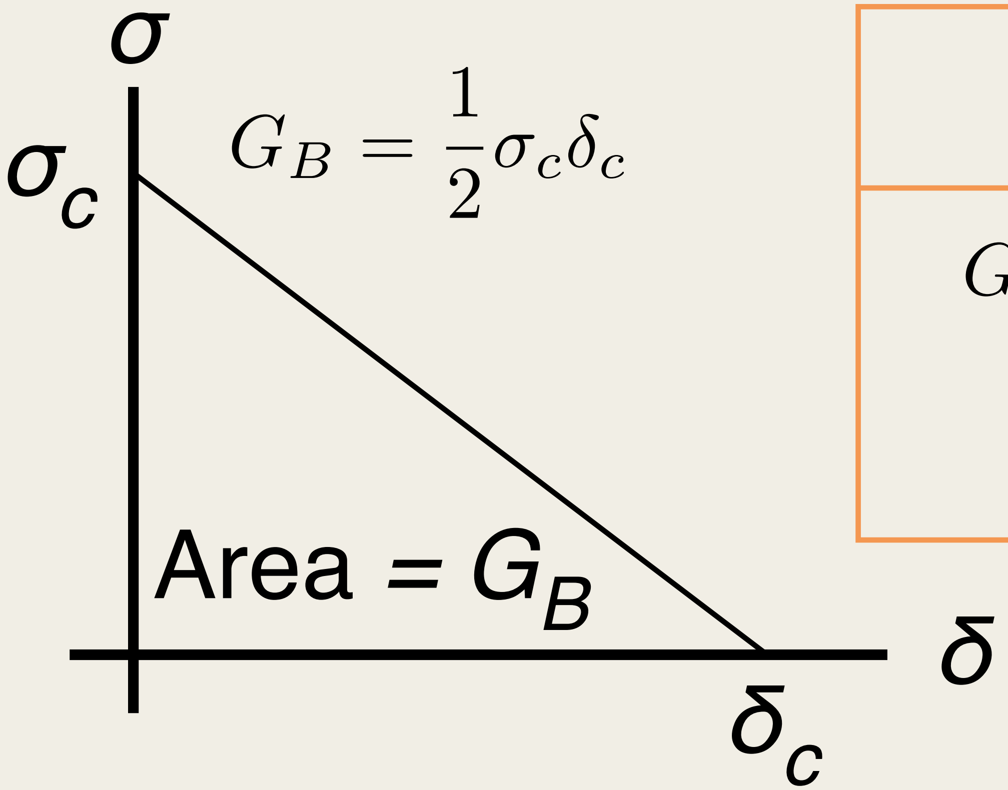


Fiber Bridging Properties

Cohesive Stress

Crack	0.75″ (MPa)	0.50″ (MPa)
38, in plane	0.43	0.79
46, in plane	0.66	2.55
38, Z cracks	0.056	0.038
46, Z cracks	0.10	0.14





In-plane bridging	Z bridging
$G_B \geq 500 - 3000\ J/m^2$	$G_B \geq 10 - 40\ J/m^2$
$\delta_c \geq 2.6\ mm$	$\delta_c \geq 0.5\ mm$

Conclusions

- Fracture with Process Zones can be Modeled well by MPM
  - Requires method that can find J_{tip} and J_{ff} from any contour
  - *R* curve modeling requires additional knowledge about the bridging zone mechanics and method to determine the amount of recoverable energy (here used elastic unloading)
  - MPM can model continuous range of models from pure fracture mechanics to pure cohesive zone model
  - No need to pre-insert cohesive zone elements
- Composite with Fiber Bridging
  - Very low cohesive stress leads to noticeable increase in toughness (and is reasonable to be low because fibers are parallel to the crack)
  - Never reaches stready-state propagation
- MDF Wood Composites
  - High in-plane toughness, very low transverse toughness (100x less)
  - Never reaches stready-state propagation
  - Can measure reasonable cohesive stress and bound the fiber bridging critical COD