Modeling drag force interaction for multi-phase flow using MPM with distinct phases

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Overview

- Introduction
- Drag Interaction of Phases (non-saturated)
- Numerical Implementation
- The locking problem
- Example
- Summary and Conclusions

Introduction : Teton Dam, ID (1976)

Introduction: Modeling Requirements

- Modeling rain-induced slides and flows requires consideration of solid and fluid phases throughout the process

- Both phases can undergo
  - mixing (wetting),
  - combined dynamic action, and
  - separation (sedimentation and drying).

- Typically, each state is characterized by specialized differential equations, thus introducing **difficulties in modeling transitions** between states.

The Vision Behind this Research

- **A unified approach for the modeling of fluids and solids**, their behavior when mixing or separating, and interaction in partially or fully saturated mixtures.

- ⇒ **Capturing the transition** from static (solid dominant) to dynamic (fluid dominant) behavior.

Modeling Framework – Constitutive Model

- **Unified approach (fluid & solid)**

  - History dependent part $$\hat{\boldsymbol{\sigma}} = \varrho \frac{\partial \psi(\boldsymbol{\varepsilon}, \boldsymbol{\xi})}{\partial \boldsymbol{\varepsilon}}$$

$$\text{with} \quad \psi(\boldsymbol{\varepsilon}, \boldsymbol{\xi}) = \bar{\psi}(dev\boldsymbol{\varepsilon}, \boldsymbol{\xi}) + \bar{U}(tr\boldsymbol{\varepsilon})$$

  - Rate dependent part $$\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}} + 2\mu \boldsymbol{\nabla}^s \mathbf{v}$$

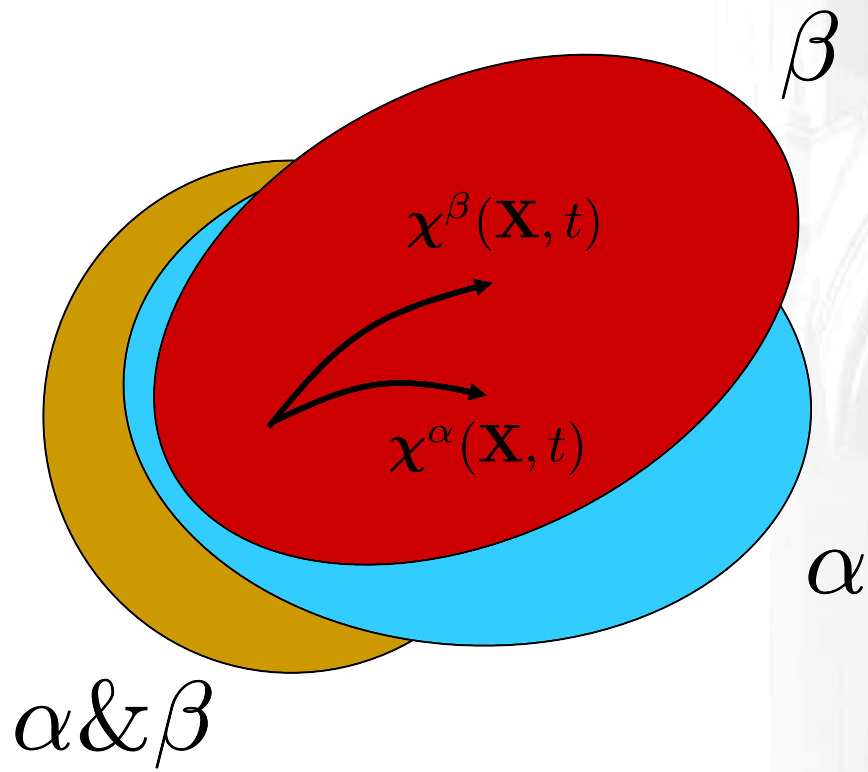
    - Pressure $$p = \varrho \frac{\partial \bar{U}(tr\boldsymbol{\varepsilon})}{\partial (tr\boldsymbol{\varepsilon})}$$

Interaction of Phase Problem

- Represent various phases α of mixtures as independent bodies which can interact if they “share a space”.

1. Track multiple distinct motions $\chi^{\alpha}(\mathbf{X}, t)$
   ⇨ multiple velocity fields
2. Identify interacting domains $V_{\alpha \cap \beta}$ and define interaction forces $\mathbf{f}^{(\alpha,\beta)}$
3. Implement volume constraint to prevent “over-saturation” of representative control volumes

Modeling Multiple Motions



- (Almost) Trivial implementation
  - *n* phases = *n* parallel analyses

Governing equations

- Variational (weak) form for a single phase

$$-\int_{m_\alpha} \bar{\boldsymbol{\sigma}}_\alpha : \boldsymbol{\eta}_\alpha \, dm_\alpha + \int_{m_\alpha} \bar{\mathbf{b}}_\alpha \cdot \boldsymbol{\eta}_\alpha \, dm_\alpha + \int_{\partial V_\alpha} \bar{\mathbf{t}}_\alpha \cdot \boldsymbol{\eta}_\alpha \, dS_\alpha - \int_{m_\alpha} \dot{\mathbf{v}}_\alpha \cdot \boldsymbol{\eta}_\alpha \, dm_\alpha = 0$$

Discrete form

$$G(\boldsymbol{\chi}_\alpha; \boldsymbol{\eta}_\alpha) \approx G^h(\boldsymbol{\chi}_\alpha; \boldsymbol{\eta}^I_\alpha) = \sum_I \boldsymbol{\eta}^I_\alpha \cdot \left(\mathbf{f}^{(\alpha)}_{\sigma,I} + \mathbf{f}^{(\alpha)}_{ext,I} - \sum_J m^{(\alpha)}_{IJ} \, \dot{\mathbf{v}}^{(\alpha)}_J \right) = 0$$

where

$$\mathbf{f}^{(\alpha)}_{\sigma,I} = -\sum_{p,\alpha} \bar{\boldsymbol{\sigma}}^{(\alpha)}_p \cdot N_I(\mathbf{x}_p) \, m^{(\alpha)}_p$$

$$\mathbf{f}^{(\alpha)}_{ext,I} = \sum_{p,\alpha} \bar{\mathbf{b}}^{(\alpha)}(\mathbf{x}_p) \, N_I(\mathbf{x}_p) \, m^{(\alpha)}_p + \int_{\partial V_\alpha} \bar{\mathbf{t}}_\alpha \, N_I(\mathbf{x}) \, dS_\alpha$$

$$m^{(\alpha)}_{IJ} = \sum_{p,\alpha} N_I(\mathbf{x}_p) N_J(\mathbf{x}_p) \, m^{(\alpha)}_p$$

Interaction forces

- Body force vs. Drag forces

$$\mathbf{b}_\alpha = \varrho_\alpha \mathbf{g} + \sum_\beta \mathbf{b}^{(\beta,\alpha)}$$

- … result of volume averaging

$$\theta^\alpha V_{RVE} = \int_{RVE} C^\alpha \, dV = \int_{V^\alpha \cap RVE} dV$$

$$\theta^\alpha \varrho^{[\alpha]} V_{RVE} = \int_{RVE} C^\alpha \varrho^{[\alpha]} \, dV = \int_{V^\alpha \cap RVE} \varrho^{[\alpha]} \, dV$$

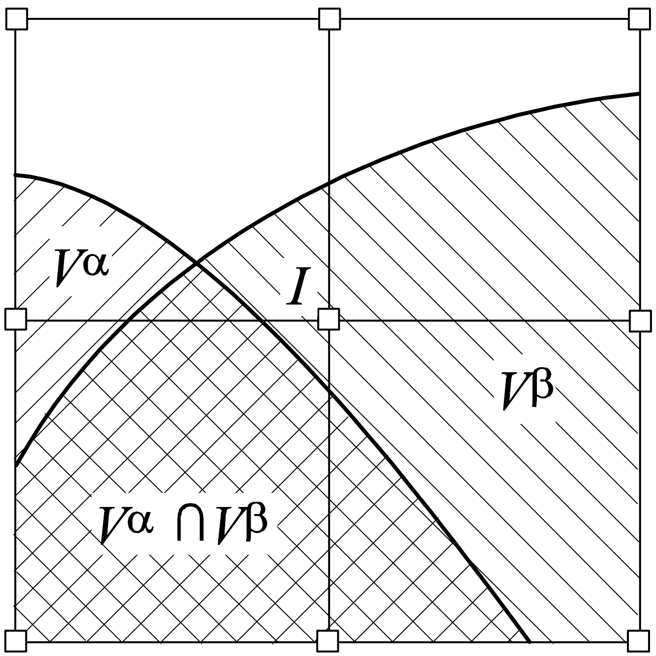
$$\theta^\alpha \varrho^{[\alpha]} \mathbf{v}^{(\alpha)} V_{RVE} = \int_{RVE} C^{[\alpha]} \varrho^{[\alpha]} \mathbf{v}^{[\alpha]} \, dV = \int_{V^\alpha \cap RVE} \varrho^{[\alpha]} \mathbf{v}^{[\alpha]} \, dV$$

$$\boxed{\theta^\alpha \theta^\beta \mathbf{b}^{(\beta,\alpha)} = \frac{1}{V_{RVE}} \int_{S^\alpha \cap S^\beta} \boldsymbol{\tau}^{[\beta,\alpha]} \, dS}$$

Interaction forces

- Drag law (averaged): $\mathbf{b}^{(\beta,\alpha)} \propto (\mathbf{v}^{(\beta)} - \mathbf{v}^{(\alpha)})$

- Nodal drag force: $\mathbf{f}_I^{(\beta,\alpha)} = \int_{V_\alpha \cap V_\beta} \mathbf{b}^{(\beta,\alpha)} N_I(\mathbf{x})\, dV_\alpha$



Nodal equilibrium

$$\mathbf{f}_I^{(\beta,\alpha)} + \mathbf{f}_I^{(\alpha,\beta)} = \mathbf{0}$$

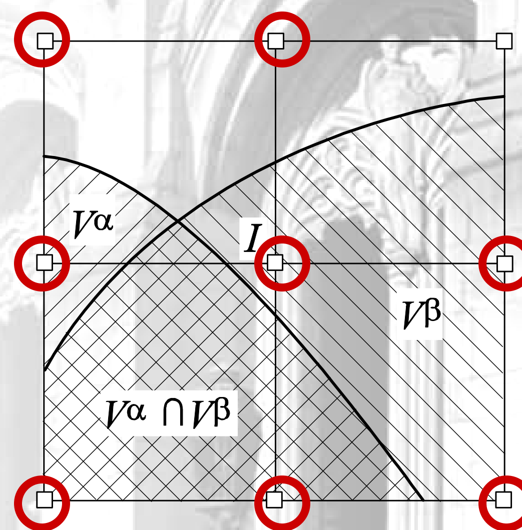
Interaction forces

- Bang-Bang Method
  - Node based

$$\mathbf{f}_I^{(\beta,\alpha)} \approx \tilde{\mathbf{b}}^{(\beta,\alpha)}(\mathbf{v}_I^{(\alpha)} - \mathbf{v}_I^{(\beta)}) \int_{V^\alpha \cap V^\beta} N_I(\mathbf{x})\, dV$$

$$\approx \tilde{\mathbf{b}}^{(\beta,\alpha)}(\mathbf{v}_I^{(\alpha)} - \mathbf{v}_I^{(\beta)}) \int_{V_{RVE}} N_I(\mathbf{x})\, dV$$

$$\boxed{\approx h^3\, \tilde{\mathbf{b}}^{(\beta,\alpha)}(\mathbf{v}_I^{(\alpha)} - \mathbf{v}_I^{(\beta)})}$$



  - Automatically satisfies $\mathbf{f}_I^{(\beta,\alpha)} + \mathbf{f}_I^{(\alpha,\beta)} = \mathbf{0}$

Interaction forces

- Bang-Bang Method
  - Particle based

$$\boxed{\mathbf{f}_I^{(\beta,\alpha)} \approx \sum_{p\in\alpha} N_I(\mathbf{x}_p^{(\alpha)})\,\tilde{\mathbf{b}}^{(\beta,\alpha)}(\mathbf{v}_\beta^h(\mathbf{x}_p^{(\alpha)}) - \mathbf{v}_p^{(\alpha)})\,\frac{m_p^{(\alpha)}}{\rho_p^{(\alpha)}}}$$

  - Nodal equilibrium requires correction

$$\Delta\mathbf{f}_I = -\tfrac{1}{2}\left(\mathbf{f}_I^{(\alpha,\beta)} + \mathbf{f}_I^{(\beta,\alpha)}\right)$$

$$\mathbf{f}_I^{(\alpha,\beta)} \quad\rightarrow\quad \mathbf{f}_I^{(\alpha,\beta)} + \Delta\mathbf{f}_I \qquad\qquad \mathbf{f}_I^{(\beta,\alpha)} \quad\rightarrow\quad \mathbf{f}_I^{(\beta,\alpha)} + \Delta\mathbf{f}_I$$

Interaction forces

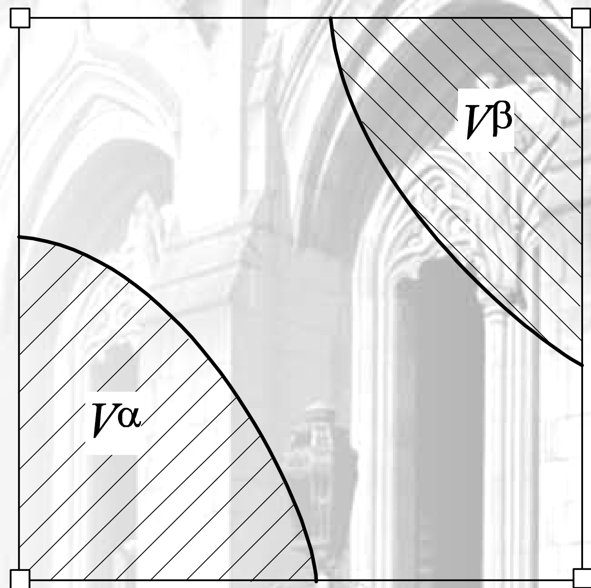
- Smooth Volume Fractions (SVF)

$$\tilde{\phi}^\alpha(\mathbf{x}) = \tilde{\phi}_0^\alpha + \nabla\tilde{\phi}^\alpha \cdot (\mathbf{x} - \mathbf{x}_c)$$

$$\int_{V_{RVE}} \tilde{\phi}^\alpha \rho^\alpha \mathbf{v}\; dV = \sum_{p\in\alpha} m_p^{(\alpha)} \mathbf{v}$$

$$\tilde{\phi}_0^\alpha = \frac{1}{V_{RVE}} \sum_{p\in\alpha\cap RVE} \frac{m_p^{(\alpha)}}{\rho_p^{(\alpha)}} \approx \frac{V^\alpha}{V_{RVE}} = \theta^\alpha$$

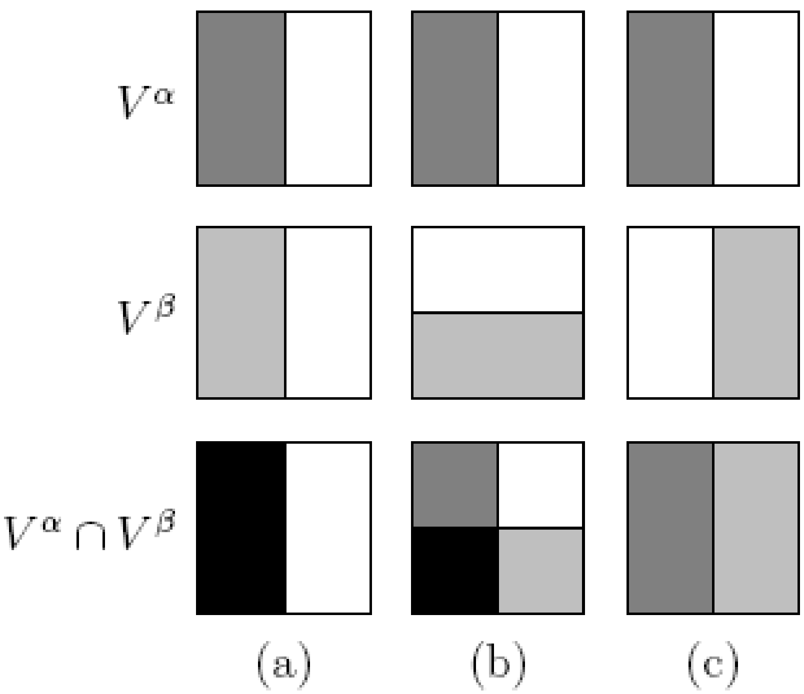
$$\nabla\tilde{\phi}^\alpha = \frac{12}{\bar{\varrho} h^4} \sum_{p\in\alpha\cap RVE} \mathbf{r}_p m_p^{(\alpha)}$$



Interaction forces

- Smooth Volume Fractions (SVF)

$$V^\alpha \cap V^\beta = \int_V \theta^\alpha \theta^\beta \, dV \approx \int_V \tilde{\phi}^\alpha \tilde{\phi}^\beta \, dV$$



	(a)	(b)	(c)
$\frac{V^\alpha \cap V^\beta}{V_{RVE}}$ (exact)	50 %	25 %	0 %
$\theta^\alpha \theta^\beta$	25 %	25 %	25 %
$\frac{\int_{RVE} \tilde{\phi}^\alpha \tilde{\phi}^\beta \, dV}{V_{RVE}}$	43.75 %	25 %	6.25 %
bang-bang methods	100 %	100 %	100 %

Interaction forces

- Smooth Volume Fractions (SVF)
  - Node based

$$\mathbf{f}_I^{(\beta,\alpha)} = \int_{V^\alpha \cap V^\beta} \tilde{\mathbf{b}}^{(\beta,\alpha)}(\mathbf{v}^{(\alpha)} - \mathbf{v}^{(\beta)}) N_I(\mathbf{x})\, dV$$

$$\approx \int_V \tilde{\mathbf{b}}^{(\beta,\alpha)}(\mathbf{v}_\alpha^h - \mathbf{v}_\beta^h) N_I(\mathbf{x})\, \tilde{\phi}^\alpha(\mathbf{x})\tilde{\phi}^\beta(\mathbf{x})\, dV$$

$$\approx \tilde{\mathbf{b}}^{(\beta,\alpha)}(\mathbf{v}_I^{(\alpha)} - \mathbf{v}_I^{(\beta)}) \int_V N_I(\mathbf{x})\, \tilde{\phi}^\alpha(\mathbf{x})\tilde{\phi}^\beta(\mathbf{x})\, dV$$

$$\boxed{\approx \tilde{\mathbf{b}}^{(\beta,\alpha)}(\mathbf{v}_I^{(\alpha)} - \mathbf{v}_I^{(\beta)})\, \tilde{\phi}_I^\alpha \tilde{\phi}_I^\beta\, h^3}$$

with

$$\tilde{\phi}_I^\alpha := \frac{\int_V N_I(\mathbf{x})\, \tilde{\phi}^\alpha(\mathbf{x})\, dV}{\int_V N_I(\mathbf{x})\, dV} \approx \frac{1}{h^3} \sum_{p\in\alpha} N_I(\mathbf{x}_p^{(\alpha)})\, \frac{m_p^{(\alpha)}}{\varrho_p^{(\alpha)}}$$

Interaction forces

- Smooth Volume Fractions (SVF)
  - Particle based

$$\boxed{\mathbf{f}_I^{(\beta,\alpha)} \approx \sum_{p\in\alpha} \tilde{\phi}^{\beta}(\mathbf{x}_p^{(\alpha)}) N_I(\mathbf{x}_p^{(\alpha)})\, \tilde{\mathbf{b}}^{(\beta,\alpha)}(\mathbf{v}_\beta^h(\mathbf{x}_p^{(\alpha)}) - \mathbf{v}_p^{(\alpha)})\, \frac{m_p^{(\alpha)}}{\rho_p^{(\alpha)}}}$$

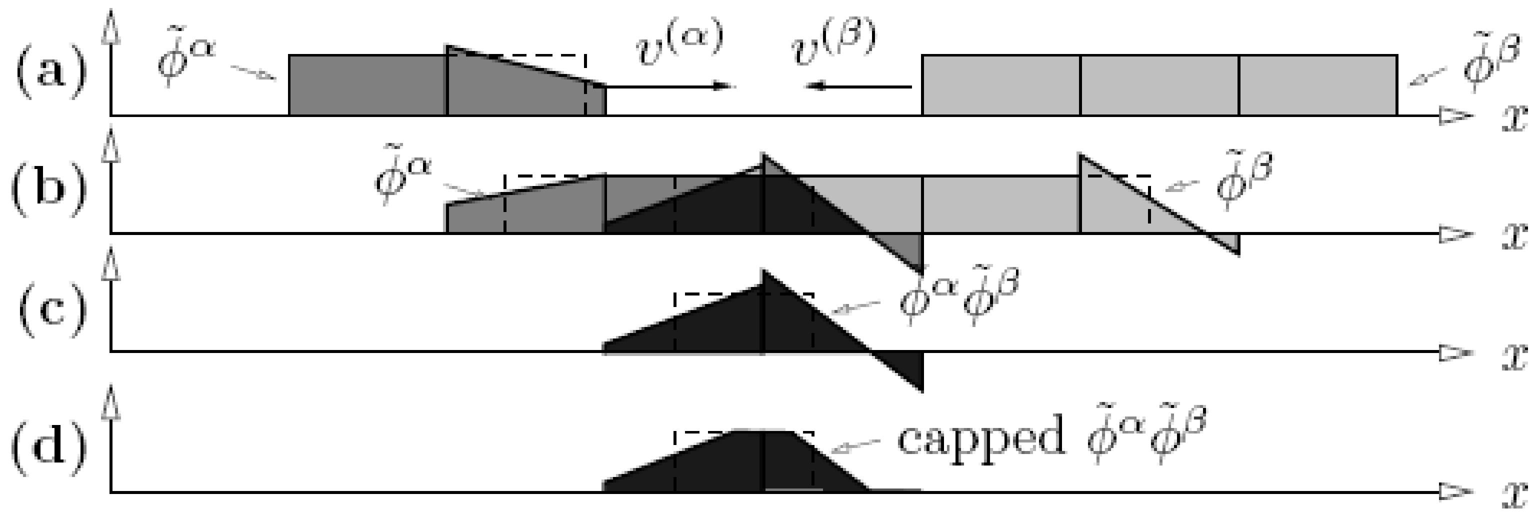
  - Nodal equilibrium requires correction

$$\Delta\mathbf{f}_I = -\tfrac{1}{2}\left(\mathbf{f}_I^{(\alpha,\beta)} + \mathbf{f}_I^{(\beta,\alpha)}\right)$$

$$\mathbf{f}_I^{(\alpha,\beta)} \quad \rightarrow \quad \mathbf{f}_I^{(\alpha,\beta)} + \Delta\mathbf{f}_I \qquad\qquad \mathbf{f}_I^{(\beta,\alpha)} \quad \rightarrow \quad \mathbf{f}_I^{(\beta,\alpha)} + \Delta\mathbf{f}_I$$

Interaction forces

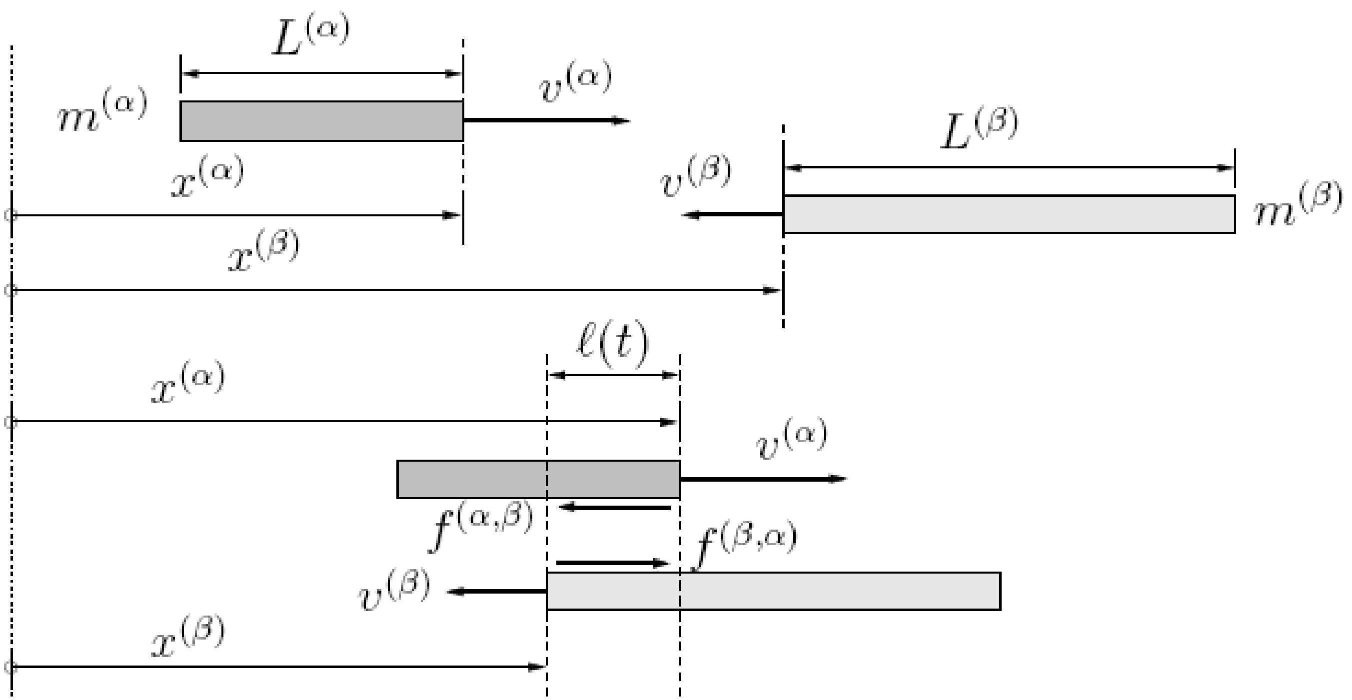
- Variations of SVF



Capped: $\boxed{0 \leq \tilde{\phi}^{\alpha}\tilde{\phi}^{\beta} \leq 1}$

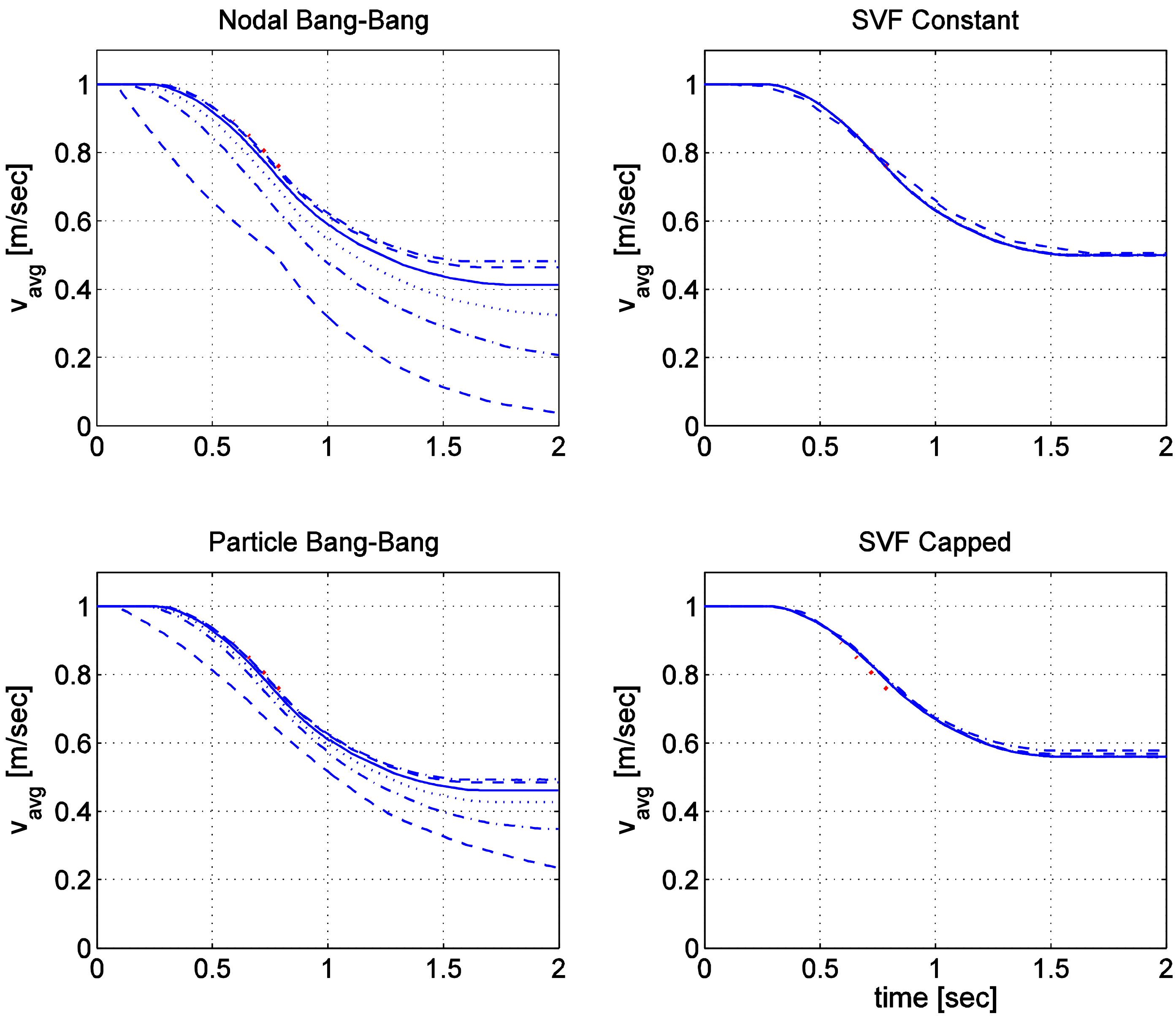
Test Cases and Error Evaluation

- Reference example



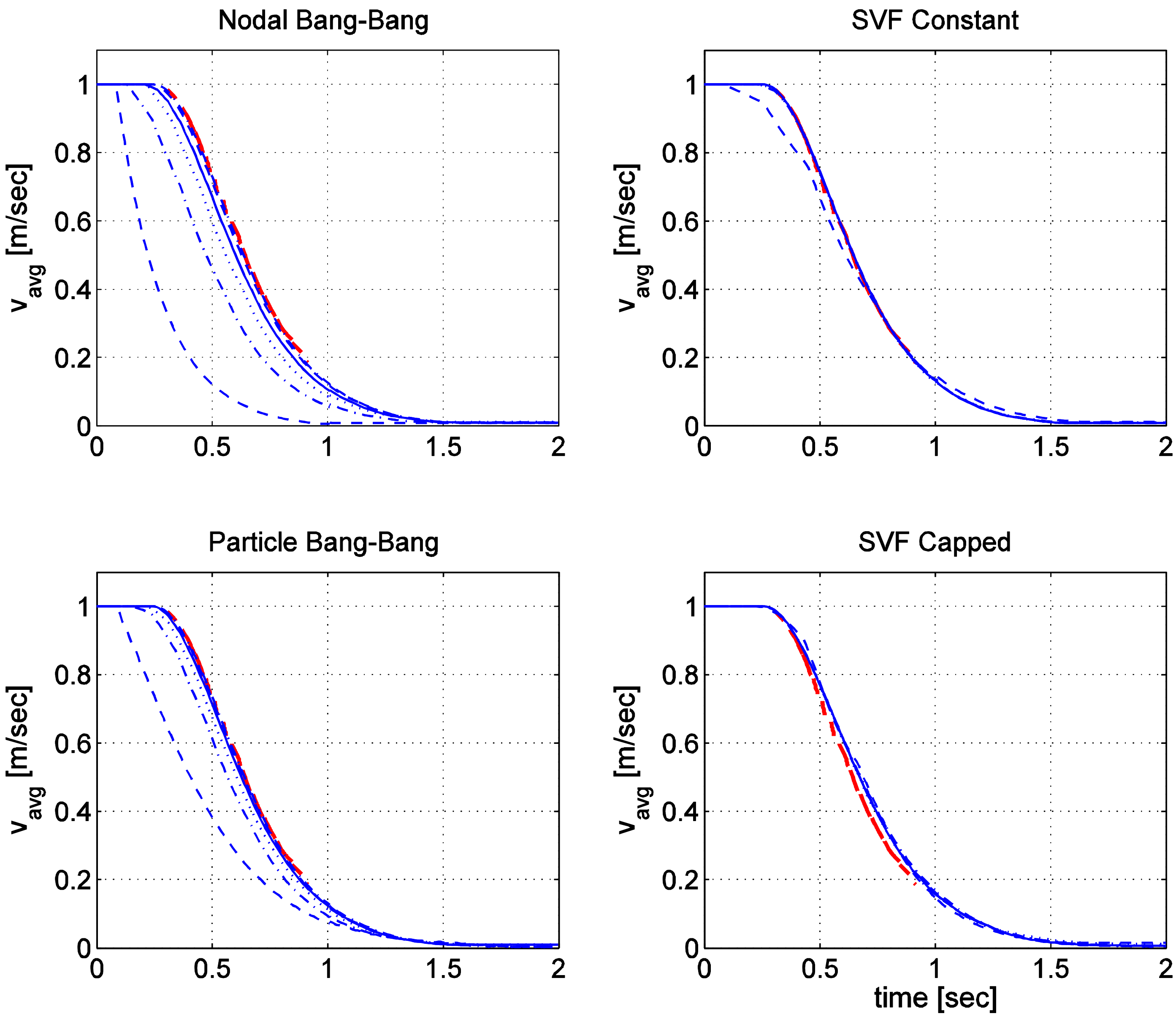
$$\mathbf{b}^{(\beta,\alpha)} = \tilde{\mathbf{b}}^{(\beta,\alpha)}(\mathbf{v}^{(\beta)} - \mathbf{v}^{(\alpha)}) := \mu\,(\mathbf{v}^{(\beta)} - \mathbf{v}^{(\alpha)})$$

Test Cases and Error Evaluation



$\mu = 1$

Test Cases and Error Evaluation

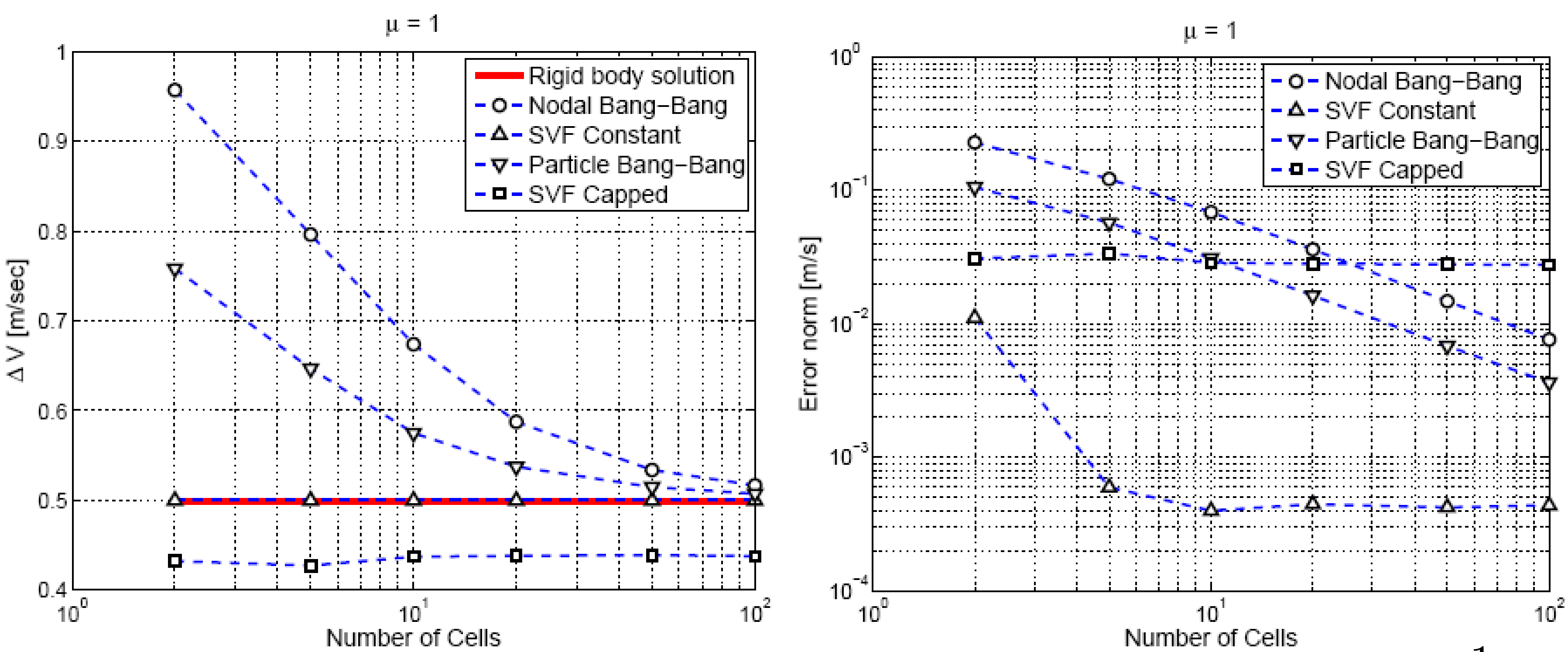


$\mu = 5$

Test Cases and Error Evaluation

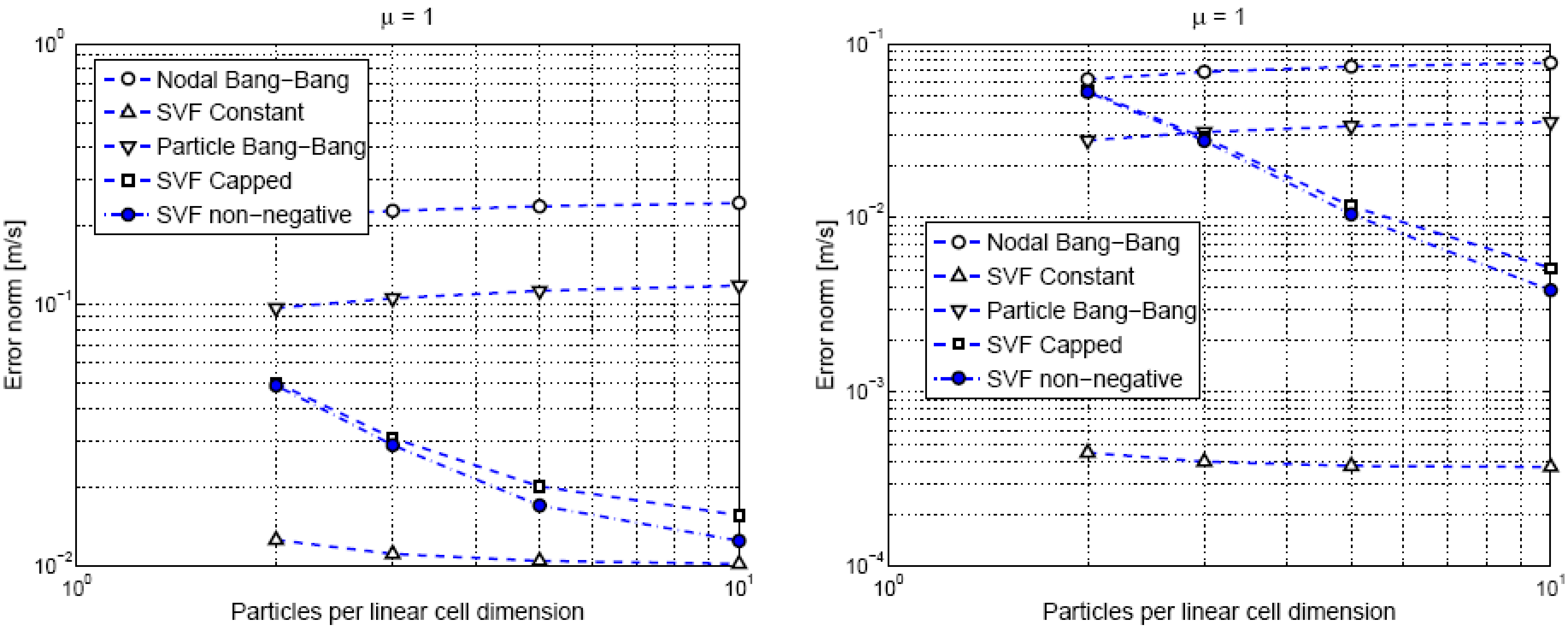
- Error assessment

$$\mathcal{E}^2 = \frac{1}{t_f} \int_0^{t_f} \left[\bar{v}(t) - v_{RB}(t)\right]^2 \, dt$$



$\mu = 1$

Test Cases and Error Evaluation



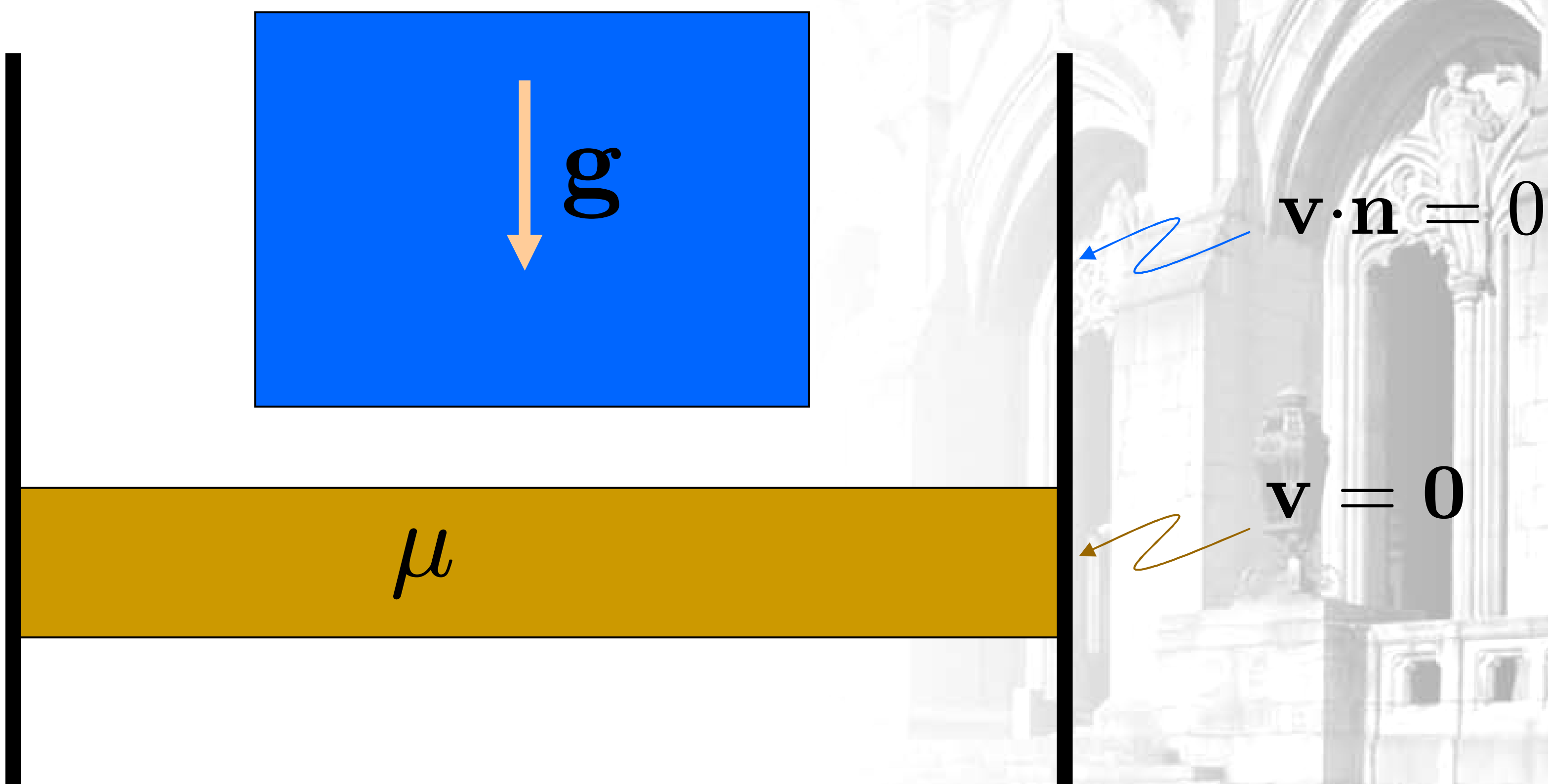
2 cells per bar length

10 cells per bar length

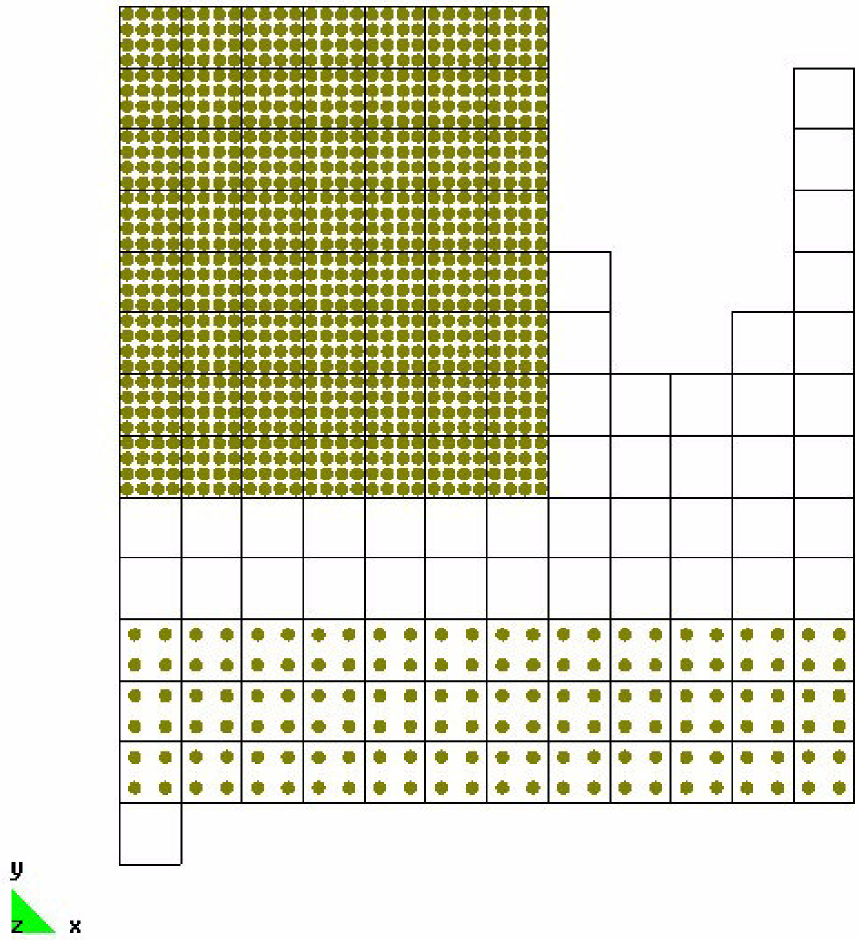
$$\mathcal{E} \leq a h^{\gamma} + b \left(\frac{d_p}{h} \right)^{\delta}$$

Test Case

- Flow through a filter stone



Test Case: Flow through a filter stone



Deformation (×1): Displacement of MY, step 0.

Locking Problem

- Representing a fluid using MPM shows unrealistic behavior at large displacements
  - Free-surface not leveling
  - Uneven penetration of fluid in filter stone

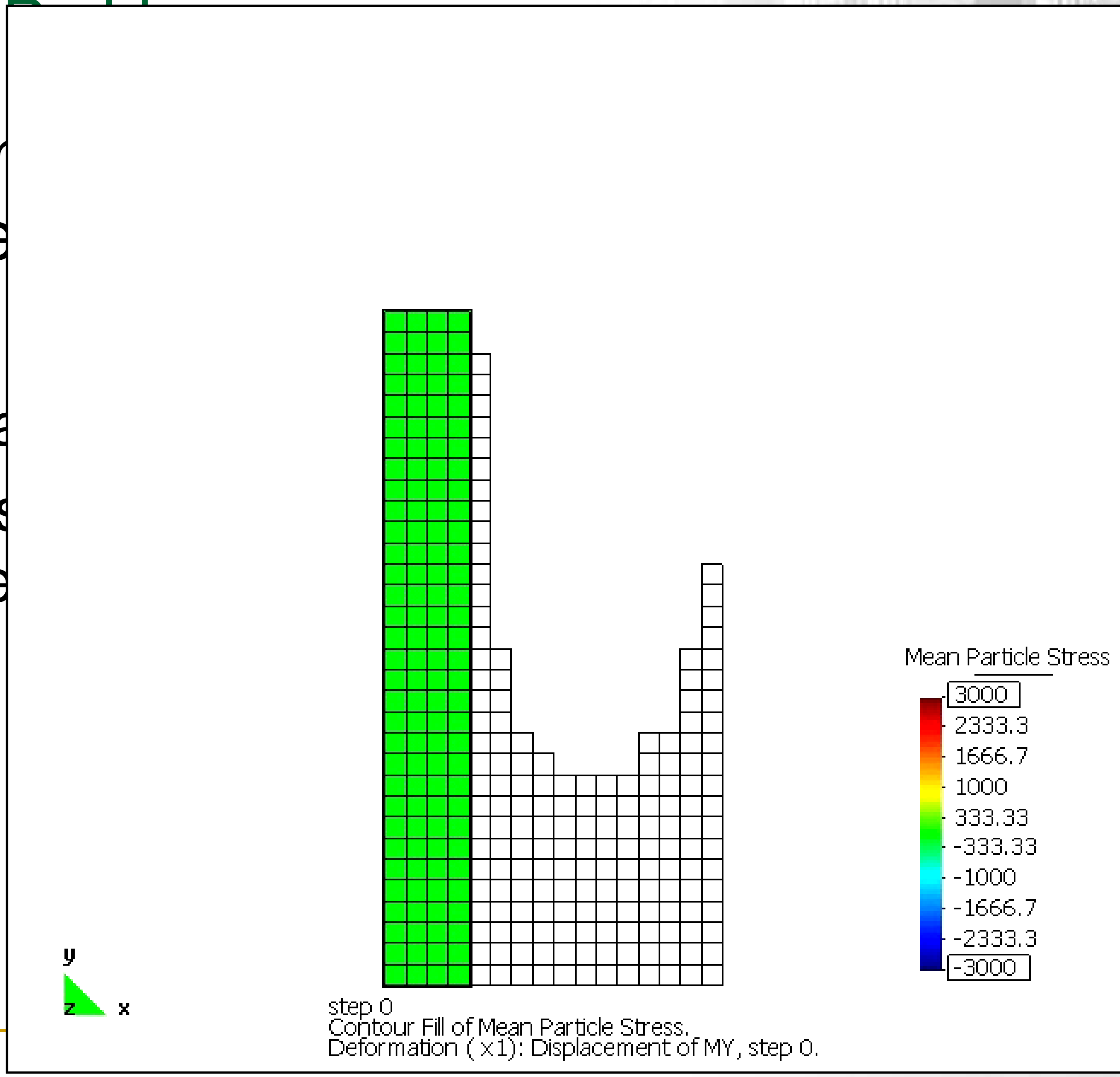
- What causes this problem?
  - General limitation on MPM ?
  - Material model ?
  - Interaction model ?

Locking Problem

- Let’s have a look at an apparently unrelated problem of MPM for solids:
  - Vibrations are induced as particles cross cells
  - Stresses are getting worse as deformations increase

Locking

- Let's h... d proble...
  - Vibra...
  - Stres... incre...



Anti-Locking Strategy

- **Source of the problem**
  - Kinematic constraints on the background grid lock-in false internal stresses
  - Fictitious internal stresses are partially released at cell crossings causing non-physical vibrations
  - Near incompressible behavior exaggerates the problem

Anti-Locking Strategy

- **Proposed solution for locking**
  - Kinematic constraints on the background grid relaxed by smoothened volume change $\bar{\theta}$

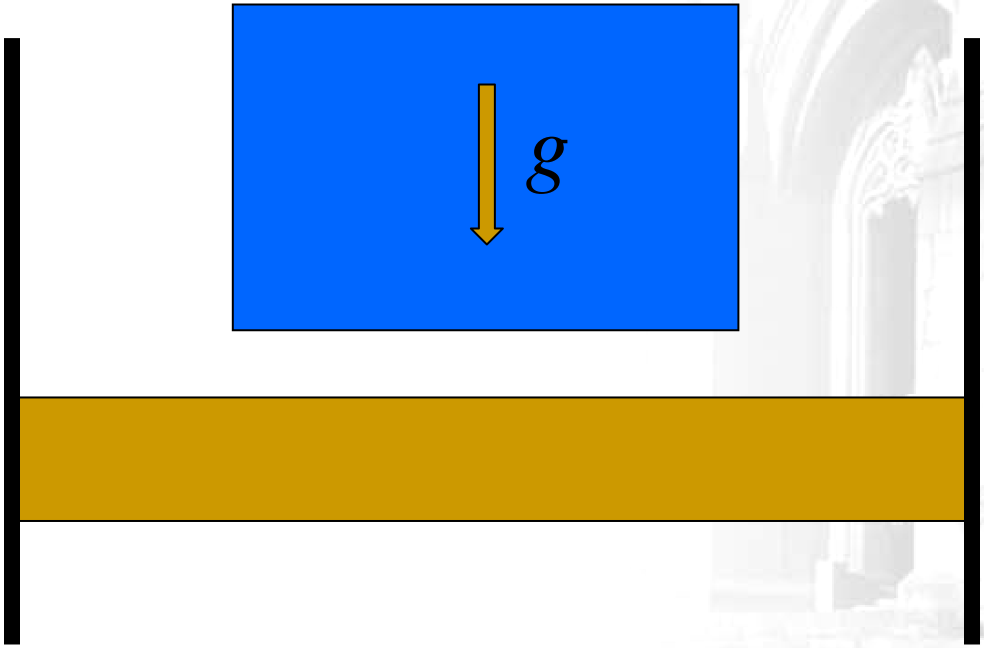
$$\int_V (\bar{\theta} - tr\boldsymbol{\varepsilon})\, \delta p \, dV = 0 \;\Rightarrow\; \boxed{\bar{\theta} = \frac{\sum_{p\in cell} tr\boldsymbol{\varepsilon}_p\, m_p/\varrho_p}{\sum_{p\in cell} m_p/\varrho_p}}$$

$$p_p = \varrho \frac{\partial \bar{U}(tr\boldsymbol{\varepsilon})}{\partial tr\boldsymbol{\varepsilon}} \;\Rightarrow\; \boxed{p_p := p_{cell} = \varrho \frac{\partial \bar{U}(\bar{\theta})}{\partial \bar{\theta}}}$$

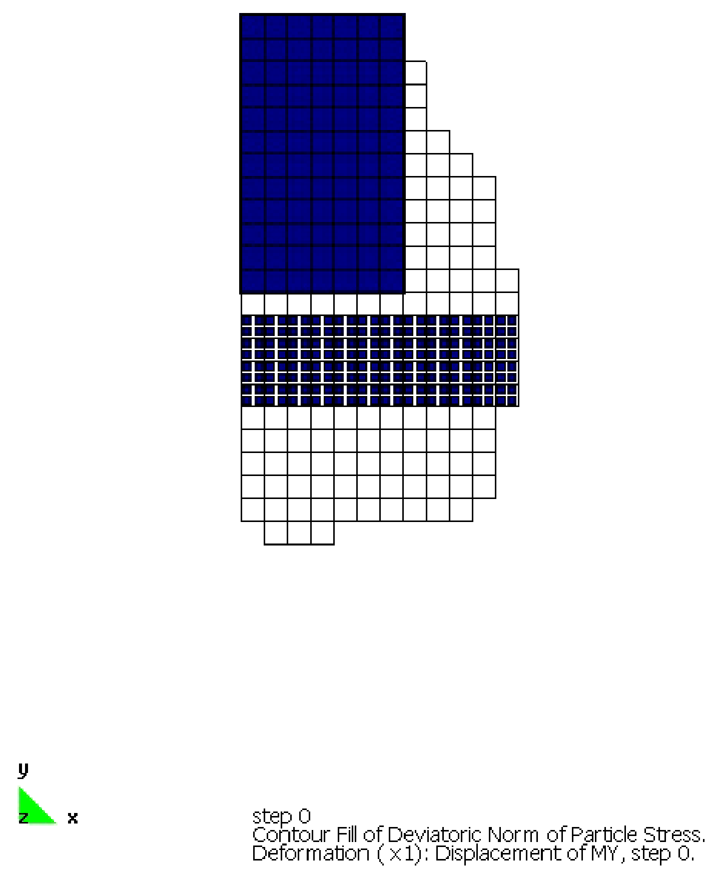
⇨ Fictitious internal stresses are relaxed at cell level

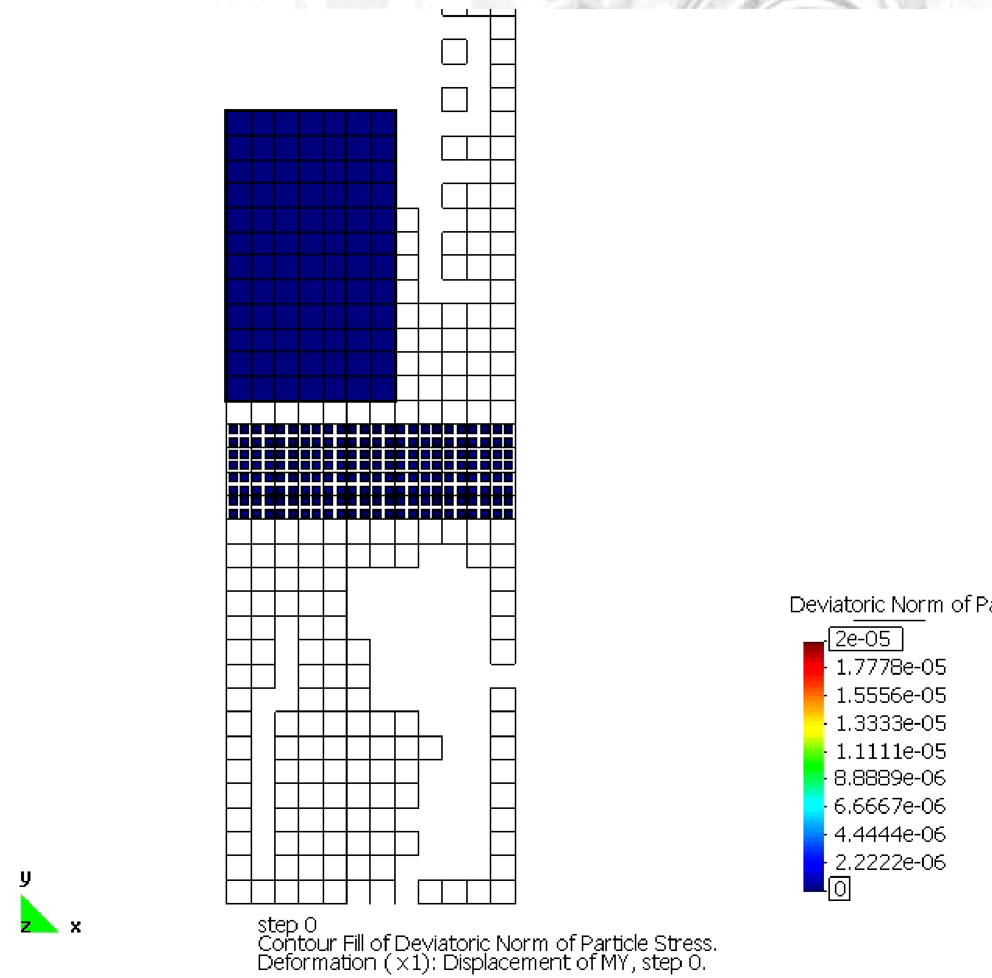
Anti-Locking Strategy

- Anti-Locking and Interaction Model
  - Filter stone problem



Anti-Locking Strategy





Summary and Conclusions

- Both bang-bang methods show a linear rate of convergence with grid refinement.
- SVF (with the exception of the SVF-capped variant) converges at least at a quadratic rate.
- SVF models are desirable over the bang-bang interaction model because they produce relatively accurate results for both a novice user, who may seek results using an unrefined grid, as well as a more experienced user who seeks to capture the crisp behavior using a small number of cells per phase.
- MPM is suitable for unified representation of solids and fluids but it requires special measures against locking (reduction of fictitious internal stresses).
- Mackenzie-Helnwein, Arduino, Shin, Moore, Miller: *Modeling Strategies for Multiphase Drag Interactions Using the Material Point Method*, submitted for publication in IJNME

Questions?

Thank you

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- This presentation went too far ☺